Thermodynamics HW #8

Due October 21

1. Schroeder 4.30. Note that you'll want to have read through the material in section 4.4 to help with the question.

   Solution:

   (a) From the table we find that 1kg of saturated vapor at 1 bar pressure has an entropy $S_1 = .940\text{kJ/K}$. Since entropy is conserved during the compression, we find by interpolation in the $P = 10$ bar row of table 4.4 the temperature at which the entropy is approximately S_1. This happens around $49°\text{C} = 322\text{K}$.

   (b) From table 4.3 we find $H_1 = 231\text{kJ}$, while an interpolation of table 4.4 gives $H_2 \approx 279\text{kJ}$. At step 3 the vapor has been completely condensed to liquid at 10 bar, so that $H_3 = 105\text{kJ}$. Point 4 is reached by a throttling process from point 3, and the analysis in Schroeder reveals that the throttling process occurs at constant enthalpy so that $H_4 = H_3$.

   From equation 4.20 we arrive at a COP of 2.63. We can compare this to the ideal COP of $\frac{T_c}{T_h - T_c}$ by looking up the temperature of the saturated vapor at 1 bar (247K) and the temperature after compression (322K).

   We know that the temperature at point 3 is less than point 2 because the enthalpy change is negative and thus heat flows *out* of the system since the process joining point 2 to point 3 is at constant temperature. We also know that the temperature at point 4 is equal to the point 1 since the points are at constant pressure and both lie inside the liquid-vapor coexistence region. Thus the ideal COP is $\frac{247}{322-247} =$ 3.3, which is larger than the actual COP, as required.

   (c) Since point 3 is 100% liquid, we begin with an enthalpy of 105kJ/kg. The enthalpy is the same at point 4, so we are then asking what fraction x of refrigerant must now be gaseous so that

   $$H_4 = x \cdot 231 + (1 - x) \cdot 16 = 105 = H_3$$

   This occurs when

   $$x = \frac{105 - 16}{231 - 16} = 41\%$$

2. Schroeder 5.4

   Solution:

   Looking ahead at section 5.2 we find that processes at constant temperature and pressure occur in the direction of decreasing Gibbs energy G.

Since the change in the Gibbs free energy is negative for the formation of water from hydrogen and oxygen, we expect that a hydrogen electrode and oxygen electrode initially at the same electric potential will execute the reaction described in the problem.

Each time the reaction occurs, however, two electrons are transferred from the hydrogen electrode to the oxygen electrode, so that an electric potential V will gradually develop between the two electrodes. Equilibrium is reached when the increase in the Gibbs energy $2e \cdot V$ balances the original the Gibbs energy, i.e.

$$V = \frac{\Delta G}{2e} = 237 \frac{\mathrm{kJ}}{\mathrm{mol}} \cdot \frac{1\mathrm{eV}}{1.6 \cdot 10^{-19}\mathrm{J}} \frac{1\mathrm{mol}}{6.02 \cdot 10^{23}\mathrm{particles}} \cdot \frac{1}{2e} = 1.23\mathrm{V}$$

Conversely, if the electric potential is sufficiently large then the Gibbs energy can become positive so that the *reverse* reaction becomes thermodynamically favorable. From the preceding we expect this to occur at an applied voltage of 1.23V.