Thermodynamics HW #5

Due September 30

1. Schroeder 2.38.

   Hint: The fact that different species A and B interact the same implies that, *if we suppose that particles of the same species are distinguishable*, we could write the multiplicity of the mixture $\Omega^{\text{dist}}_{A\oplus B}$ in the following way:

   $$\Omega^{\text{dist}}_{A\oplus B}(N_A, N_B) = \tilde{\Omega}(N_A + N_B)$$

   So that the multiplicity would only depend on the total number of particles and not how they were divied up between species A and species B. Your job is to modify $\Omega^{\text{dist}}_{A\oplus B}$ to account for the indistinguishability of particles of the same species.

2. Suppose we have a rigid insulated container with a internal wall dividing the volume into two regions, #1 and #2. Region #1 contains a monotonic ideal gas with energy U_1, volume V_1 and particle number N_1 not necessarily equal to the energy U_2, volume V_2, and particle number N_2 of region #2. For each of the following processes, determine the condition(s) necessary for the process to be irreversible:

   (a) The dividing wall is fixed and can not transmit particles but can transmit heat. The process entails a transfer of some small amount of energy dU from region #2 into region #1.

   (b) The dividing wall can not transmit particles but can move (like a piston) and can transmit heat. Assume that the wall is a good conductor so that at all times the temperatures of the two gases are equal. The process entails a displacement of the wall so that there is a small transfer of volume dV from region #2 into region #1.

   (c) The dividing wall is fixed but conductive to heat and has a hole in it so that particles can travel between the two regions. Assume the two regions have equal temperature at all times. The process entails a transfer of some small number of particles dN from region #2 into region #1.

   Hint: The key is to figure out how all the variables must change in order to satisfy the contraints. For example, the insulating walls of the rigid container prevent any exchange of energy between the container and its surroundings. That means if region #1 increases in energy by $dU_1 \equiv dU$ that there must a corresponding *decrease* in energy so that $dU_2 = -dU$. Likewise, a fixed wall implies no change in the volumes of either region so that $dV_1 = 0$ and $dV_2 = 0$. What condition is imposed by the inability of the wall to transmit particles? What does the rigidity of the container imply about an increase of volume dV in region #1? Increase in particle number dN? The questions as they are specified allow for a determination of the all the other differentials in terms of the given differentials (i.e. dU for part a), dV for part b), dN for part c)).

   Answers: **a)** $T_1 = T_2$, **b)** $P_1 = P_2$, **c)** $\frac{S_1}{N_1} = \frac{S_2}{N_2}$