Thermodynamics HW #4

Due September 23

1. Schroeder 2.8

2. Let A and B be two systems (not necessarily Einstein solids) and let $\Omega_A(U_A)$ and $\Omega_B(U_B)$ be the respective number of microstates available to each system (i.e. system A has $\Omega_A(U_A)$ available microstates when it has an energy U_A and likewise for B).

   Show that the A and B as a combined system are at equilibrium when $\frac{d}{dU_A}\ln\Omega_A = \frac{d}{dU_B}\ln\Omega_B$.

   In light of this observation, what property might be we tempted to associate with the functions $\frac{d}{dU_A}\ln\Omega_A$ and $\frac{d}{dU_B}\ln\Omega_B$?

3. Investigate the equilibrium condition established in the previous problem using a system consisting of two Einstein solids A and B, where $N_A = 5$ and $N_B = 15$, and $q = 30$.

   (a) Plot $\Omega_A(q_A)\Omega_B(q - q_A)$ vs. q_A for $q_A = 0, 1, 2, \ldots, q$ and identify the value q_A^{MAX} for which the plot is maximum. For ease of viewing you will want to plot the y-axis on a log scale.

   (b) Now construct a different plot with two curves:

   - $\frac{\ln\Omega_A(q_A+1) - \ln\Omega_A(q_A-1)}{2}$, and
   - $\frac{\ln\Omega_B(q-q_A+1) - \ln\Omega_B(q-q_A-1)}{2}$

   for $q_A = 1, 2, \ldots, q - 1$.

   Where do you expect the curves to intersect? Why? Was your expectation correct?

   Now suppose N_B and q grow very large while N_A stays small. (If it helps you may also assume $q \gg N_B$.) In this case, how do you expect $\frac{d}{dU_A}\ln\Omega_A(q_A^{\mathrm{MAX}})$ to change if you double the size of system A? Why?

4. Repeat the derivation of Schroeder equation (2.28), this time for the low temperature limit ($q \ll N$). *Hint: note the symmetry of the right-hand-side of equation (2.17). You should be able to immediately write down the low temperature analog of equation (2.21) and go from there.*