Thermodynamics HW #3

Due September 16

1. Schroeder 1.41
2. Schroeder 1.45 a),b)
3. Schroeder 1.49
4. Schroeder 1.50 a),b),c),d),e) and replace f) with the following question:

   Suppose the reaction is carried out in a vessel with a piston so that the pressure is constant and equal to the surrounding pressure but the walls are adiabatic so that no heat is transferred to the surroundings. What is the enthalpy change of the gas in the vessel as a result of the reaction?

   Bonus: What if the walls are adiabatic and the volume is fixed (no piston)? What is the change in energy as a result of the reaction? Where did the chemical energy go?

5. Schroeder equation (2.6) gives the number of outcomes from flipping a coin N times that result in obtaining n heads. If we presume that all outcomes are equally likely, then by dividing the right-hand side of equation (2.6) by total number of outcomes 2^N we obtain the *probability* $P_N(n)$ that N coin flips results in n heads. This distribution is known as the *binomial* distribution.

   (a) Prove that the mean number of heads is $N/2$, as expected. In other words show that

   $$\langle n \rangle_N \equiv \sum_{n=0}^{N} P_N(n) n = \frac{N}{2}$$

   (b) Next prove that the expectation value of n^2 is $\frac{N(N+1)}{4}$, i.e. show that

   $$\langle n^2 \rangle_N \equiv \sum_{n=0}^{N} P_N(n) n^2 = \frac{N(N+1)}{4}$$

   and use the identity

   $$\sigma_N = \sqrt{\langle n^2 \rangle_N - \langle n \rangle_N^2}$$

   to show that we get for the standard deviation

   $$\sigma_N = \frac{\sqrt{N}}{2}$$

   (c) Chebyshev's inequality states that the probability of a random variable lying more than k standard deviations away from the mean value is no greater than $1/k^2$. Use this inequality, along with the results from a) and b) to give an upper bound on the probability of finding more than 51% or less than 49% of the air in a box in the right half of a box. Assume the box contains 10^{24} gas particles.