10.6

COMPARING 10.48 & 10.34, WE FIND

$$\delta_\ell = \operatorname{Arcsin}\left[\frac{j_\ell(ka)}{h_\ell^{(1)}(ka)}\right]$$

(WITHIN A SIGN)

10.9

$$2E_1 \frac{a}{r}$$

$$-\frac{m}{2\pi\hbar^2}\int \frac{e^{ik|\vec r-\vec r_0|}}{|\vec r-\vec r_0|}\left[\frac{(-e^2)}{4\pi\varepsilon_0 r_0}\right]\frac{1}{\sqrt{\pi a^3}}e^{-r_0/a}\, r_0^2 dr_0 \cdot \sin\theta \cdot d\theta \cdot d\phi$$

$$k \equiv iK \equiv i\left[-2mE\right]^{1/2}/\hbar = i\left[2m\frac{1}{2m}\left(\frac{\hbar}{a}\right)^2\right]^{1/2}/\hbar$$

[SETTING E TO E_1, GROUND STATE ENERGY]

$$= i/a$$

$$= \frac{1}{\sqrt{\pi a^3}}\,\frac{-m}{2\pi\hbar^2}\,2E_1 a^2\int z_0 dz_0\, e^{-z_0}\cdot \sin\theta\, d\theta\, d\phi \cdot e^{-|\vec z-\vec z_0|}/|\vec z-\vec z_0|$$

$$[\vec z \equiv \vec r/a,\ \vec z_0 \equiv \vec r_0/a$$

ASSUME W/OUT LOSS OF GENERALITY THAT

$$\vec z = z\hat z \quad \text{SO} \quad |\vec z-\vec z_0| = \left(z^2+z_0^2-2zz_0\cos\theta\right)^{1/2}$$

ϕ - INTEGRAL THEN GIVES 2π AND

$$= \frac{1}{\sqrt{\pi a^3}}\left(\frac{-m}{\hbar^2}\right)2E_1 a^2\int_0^\infty dz_0\, z_0 e^{-z_0}\int_0^\pi d\theta\, \frac{e^{-z'}}{z'}\sin\theta\, d\theta$$

$$\left(E_1 = -\hbar^2/2ma^2\right] \qquad z' \equiv |\vec z - \vec z_0|$$

θ - INTEGRAL

$$dz' = \frac{-1}{z} \cdot \frac{1}{z'} \cdot 2 z z_0 \sin\theta \, d\theta$$

$$= - z z_0 \frac{\sin\theta \, d\theta}{z'}$$

or: $$-\frac{dz'}{z z_0} = \frac{\sin\theta \, d\theta}{z'}$$

$$\Rightarrow \int_0^\pi d\theta \, e^{-z'} \sin\theta \, d\theta / z' = -\frac{1}{z z_0} \int_{|z-z_0|}^{|z+z_0|} e^{-z'} dz'$$

$$= \frac{1}{z z_0} \left[e^{-(z+z_0)} - \begin{cases} e^{-z+z_0} & z_0 < z \\ e^{+z-z_0} & z_0 > z \end{cases} \right]$$

RADIAL INTEGRAL

$$\cancel{\frac{1}{\sqrt{\pi a^3}} \frac{z}{z} \int_0^a e^{-z_0} \frac{dz_0}{z} \left[e^{-z-z_0} - \begin{cases} e^{-z+z_0} & z_0 < z \\ e^{z-z_0} & z_0 > z \end{cases} \right]}$$

$$\cancel{= \frac{1}{\quad}}$$

$$= \frac{1}{\sqrt{\pi a^3}} \frac{1}{z} \int_0^a e^{-z_0} dz_0 \left[e^{-z-z_0} - \begin{cases} e^{-z+z_0} & z_0 < z \\ e^{z-z_0} & z_0 > z \end{cases} \right]$$

THE FIRST TERM IS:

$$\frac{1}{\sqrt{\pi a^3}} \frac{e^{-z}}{z} \int_0^{\infty} e^{-2z_0} = \frac{1}{\sqrt{\pi a^3}} \frac{e^{-z}}{2z} e^{-2z_0} \Big|_{\infty}^{0}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{\pi a^3}} \frac{e^{-z}}{z}$$

THE SECOND TERM IS:

$$= \frac{-1}{\sqrt{\pi a^3}} \frac{1}{z} \left(e^{-z} \int_0^z \cancel{dz_0} + e^{z} \underbrace{\int_z^{\infty} e^{-2z_0}\, dz_0}_{} \right)$$

$$\frac{1}{2} e^{-2z_0} \Big|_{\infty}^{z}$$

$$= \frac{1}{2} e^{-2z}$$

$$= \frac{-1}{\sqrt{\pi a^3}} e^{-z} - \frac{1}{\sqrt{\pi a^3}} \frac{1}{2} \frac{e^{-z}}{z}$$

PUTTING IT ALL TOGETHER, THE 2^{ND} TERM ON THE RHS OF EQN 10.67 IS:

$$\frac{-1}{\sqrt{\pi a^3}} e^{-r/a}$$

, SO I PRESUME I'VE MADE A ~~S~~ SIGN ERROR SOMEWHERE WHICH WE'LL IGNORE :-). IF WE SET $\psi_0 = 0$ WE HAVE SOLVED THE PROBLEM.

10.10

From Eq$^{\underline{n}}$ 10.88:

$$f(\theta) \simeq -\frac{2m}{\hbar^2 k}\int_0^a dr\, r\, V_0 \sin(kr)$$

$$= -\frac{2m}{\hbar^2}\cdot\frac{V_0}{k^3}\int_0^{z_0} dz\, z \sin(z)$$

$$(z_0 \equiv ka)$$

$$dz\, z \sin z = \cos z\, dz - d\left[z\cos z\right]$$

$$= d\left[\sin z - z\cos z\right]$$

$$\Rightarrow f(\theta) \simeq -\frac{2m}{\hbar^2}\cdot\frac{V_0}{k^3}\left[\sin z_0 - z_0\cos z_0\right]$$

@ low energy $z_0 \to 0$

$$\Rightarrow \sin z_0 - z_0 \cos z_0 \longrightarrow$$

$$\left[z_0 - \frac{1}{6}z_0^3 + \cdots\right] - z_0\left[1 - \frac{1}{2}z_0^2 + \cdots\right]$$

$$= \frac{1}{3}z_0^3 + \cdots$$

$$\Rightarrow f(\theta) \longrightarrow -\frac{2mV_0}{\hbar^2}\frac{a^3}{3} = -\frac{m}{2\pi\hbar^2}V_0\left(\frac{4}{3}\pi a^3\right)$$

10.13

a) EQ^N 10.88

b)

c) $$f(\theta) = -\frac{2m}{\hbar^2 K}\int_0^\infty \alpha\,\delta(r-a)\,r\,dr\,\sin(Kr)$$

$$= -\frac{2m\alpha}{\hbar^2 K}\,a\sin(Ka)$$

$$= -\frac{2m\alpha a}{\hbar^2 2k\sin\theta/2}\,\sin(2ka\sin\theta/2)$$

@ LOW ENERGY $k \to 0$

$$f(\theta) \longrightarrow -\frac{2m\alpha a^2}{\hbar^2}$$

$$D(\theta) = |f|^2 = \left[\frac{2m\alpha a^2}{\hbar^2}\right]^2$$

$$\sigma = \int d\Omega\, D(\theta) = 4\pi D(\theta) = 4\pi\left[\frac{2m\alpha a^2}{\hbar^2}\right]^2$$

BORN-APPROX IS A WEAK POTENTIAL APPROX, SO TAKING $\alpha \to 0$ IN ANSWER TO 10.4:

$$\sigma = 4\pi a^2 \cdot \frac{\beta^2}{[1+\beta]^2} \longrightarrow 4\pi\left[\frac{2m\alpha a^2}{\hbar^2}\right]^2$$

$\to 0$ $[\beta \overset{\text{PROP TO}}{\propto} \alpha]$

✓