11.17 a)

$$V_{if} = \frac{1}{l^{3/2}} \cdot \frac{1}{\pi^{1/2}} \cdot \frac{1}{a^{3/2}} \int d\tau \, e^{i\vec{k}\cdot\vec{r}} \, e^{-r/a} \, [+E_0 e z]$$

AS GRIFFITHS POINTS OUT:

$$z e^{i\vec{k}\cdot\vec{r}} = -i \frac{d}{dk_z} e^{i\vec{k}\cdot\vec{r}}$$

SO THAT, NEGLECTING FACTORS OF UNIT LENGTH:

$$V_{if} = \frac{E_0 e}{[l^3 a^3 \pi]^{1/2}} \frac{d}{dk_z} \int e^{i\vec{k}\cdot\vec{r}} \, e^{-r/a} \, d\tau$$

THE INTEGRAL IS IDENTICAL IN FORM TO 10.79, WHERE WE SUBSTITUTE $\vec{k}' - \vec{k} \rightarrow \vec{k}$ AND $V(\vec{r}_0) \rightarrow e^{-r/a}$.

SINCE $e^{-r/a}$ IS SPHERICALLY SYMMETRIC, THE ANALYSIS IN SECTION 10.4.2 CAN BE REUSED AND THE INTEGRAL SIMPLIFIES TO

$$\frac{4\pi}{k} \int_0^\infty r e^{-r/a} \sin(kr) \, dr$$

NOTE THAT:

$$\int_0^\infty r e^{-r/a} \sin kr\, dr$$

$$= \int_0^\infty d\left[\frac{-r}{a}\right][-a] \cdot \left(\frac{-r}{a}\right)[-a] \cdot e^{-r/a} \cdot \sin\left([-a]k\left[\frac{-r}{a}\right]\right)$$

$$= a^2 \int_{-\infty}^{0} x\, dx\, e^x \sin\beta x \qquad [\beta \equiv ka]$$

AND:

$$\sin\beta x\, e^x\, dx = \sin\beta x\, d\left[e^x\right]$$

$$= d\left[e^x \sin\beta x\right] + \beta e^x \cos\beta x\, dx$$

$$= d\left[e^x \sin\beta x\right] - \beta \cos\beta x\, d\left[e^x\right]$$

$$= d\left(e^x \sin\beta x\right) - \beta\left(d\left[e^x \cos\beta x\right] + \beta e^x \sin\beta x\, dx\right)$$

$$= d\left(e^x\left[\sin\beta x - \beta\cos\beta x\right]\right) - \beta^2 e^x \sin\beta x\, dx$$

SO

$$e^x \sin\beta x\, dx = \left(1+\beta^2\right)^{-1} d\left[e^x\left(\sin\beta x - \beta\cos\beta x\right)\right]$$

So $x e^x \sin\beta x \, dx$

$$= \left[1+\beta^2\right]^{-1} x \, d\left[e^x\left[\sin\beta x - \beta\cos\beta x\right]\right]$$

$$= \left[1+\beta^2\right]^{-1}\left[\underbrace{d\left(x e^x\left(\sin\beta x - \beta\cos\beta x\right]\right)}_{\text{VANISHES @ ENDPOINTS } \{0,\,-\infty\}}\right.$$

$$\left. - e^x\left(\sin\beta x - \beta\cos\beta x\right] dx\right]$$

$$= \left(1+\beta^2\right]^{-1} e^x\left(\beta\cos\beta x - \sin\beta x\right] dx$$

$$= \left(1+\beta^2\right]^{-1}\left[\beta\left(d\left[e^x\cos\beta x\right] + \beta e^x \sin\beta x \, dx\right]\right.$$

$$\left. - \sin\beta x \, dx\right]$$

$$= \left(1+\beta^2\right]^{-1}\left(\beta \, d\left[e^x\cos\beta x\right]\right.$$

$$\left. + \left(\beta^2 - 1\right] e^x \sin\beta x \, dx\right)$$

$$= \left(1+\beta^2\right)^{-1}\left(\beta \, d\left(e^x\cos\beta x\right]\right.$$ VANISHES @ ENDPOINTS

$$\left. + \frac{\left(\beta^2-1\right)}{\left(1+\beta^2\right)} \, d\left(e^x\left(\overset{\sin\beta x}{\cancel{\beta\cos\beta x}} - \beta\cos\beta x\right]\right]\right)$$

$$= \left(1+\beta^2\right)^{-2}\left(\left[1+\cancel{\beta^2}+1-\cancel{\beta^2}\right]\beta \, d\left(e^x\cos\beta x\right]\right)$$

$$= \frac{2\beta}{(1+\beta^2)^2} \, d\left[e^{x} \cos\beta x \right]$$

so Yitray :

$$\int_{-\infty}^{0} dx \; x \, e^{x} \sin\beta x$$

$$= \frac{2\beta}{(1+\beta^2)^2} \underbrace{\int_{-\infty}^{0} d\left[e^{x} \cos\beta x \right]}_{=1}$$

$$= 2\beta \left(1+\beta^2 \right)^{-2}$$

AND $\beta = ka = \left(k_x^2 + k_y^2 + k_z^2 \right)^{1/2} a$

so $\frac{d}{dk_z}\beta = a \cdot \frac{1}{2} \cdot \frac{1}{k} \cdot 2k_z$

$= a\cos\theta$

so

$$\frac{d}{dk_z}\left(\frac{2\beta \cdot 4\pi \frac{a}{\beta}}{\left[1+\beta^2\right]^2} \right) = \frac{d}{d\beta}\left(\frac{2\beta 4\pi a}{\left[1+\beta^2\right]^2} \right) \cancel{\frac{d\beta}{dk_z}} \; a\cos\theta$$

$$= a\cos\theta \left(\frac{2}{(1+\beta^2)^2} - \frac{2}{(1+\beta^2)^3} \cdot 2\beta \right)$$

$$\frac{d}{dk_z}\left[\int e^{i\vec{k}\cdot\vec{r}}\, e^{-r/a}\, d\tau\right]$$

$$= \frac{d}{dk_z}\left[\frac{4\pi}{k}\cdot\frac{2\beta a^2}{(1+\beta^2)^2}\right]$$

$$= \frac{d}{dk_z}\left(8\pi a^3\cdot(1+\beta^2)^{-2}\right)$$

$$= 8\pi a^3 \frac{d}{d\beta}(1+\beta^2)^{-2}\ \frac{d}{dk_z}\beta \xrightarrow{} a\cos\theta$$

$$= 8\pi a^4 \cos\theta\left[-2(2\beta)(1+\beta^2)^{-3}\right]$$

$$= 32\pi a^4\cos\theta\ \beta\,(1+\beta^2)^{-3}$$

The rest is plugging into Eq^{n} 11.81:

$$R = \cancel{\frac{2\pi}{\hbar}}\left|\frac{32\,\pi\, a^{k^{5/2}}}{\cos\theta}\,\beta\,(1+\beta^2)^{-3}\ \cdot\frac{\cos\theta}{\cancel{e^{3/2}}\,\cancel{\pi^{1/2}}\,\cancel{a^{3/2}}}\right|^2$$

$$e^2E_0^2\ 256\ \frac{32\,\pi\, e^3\,(2\pi)}{4\pi^2\,(2\pi)^{3/2}}\left(\frac{2m\,\hbar^2 k^2}{2m}\right)^{1/2}\, d\Omega\ m k\cos^2\theta$$

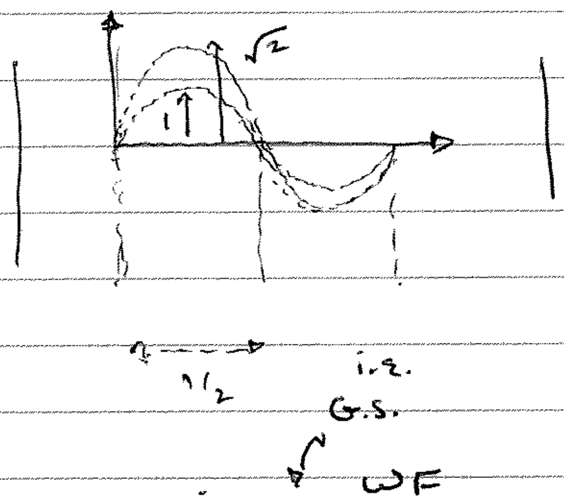
$$\hbar^3$$

= ...

11.18

a) OVER THE REGION WHERE THE STATE IS NON-ZERO, THE 2^{ND} W.F. OF THE NEW BOX IS EXACTLY PROPORTIONAL TO THE STATE AND SO ITS INNER PRODUCT [AND THUS PROBABILITY] IS MAXIMIZED.

CONVINCE YOURSELF THAT THE ~~b)~~ ~~TH~~ PROBABILITY MUST BE 1/2 :



$$= \left| 1/\sqrt{2} \right|^2 = 1/2$$

b) THE FIRST ~~STATE~~ IS THE ONLY OTHER STATE TO NOT HAVE FULL OR PARTIAL CANCELLATION SO QUALITATIVELY EASY TO SEE THAT IT HAS NEXT ~~HIGHEST~~ PROBABILITY.

$$P_1 = \frac{2}{a^2} \left| \int_0^a \sin\left(\frac{\pi x}{2a}\right) \sin\left(\frac{2\pi x}{2a}\right) dx \frac{\pi}{2a} \frac{2a}{\pi} \right|^2$$

$$= \frac{8}{\pi^2} \left| \int_0^{\pi/2} \sin x \sin 2x \, dx \right|^2$$

$$= \frac{8}{\pi^2} \left(\int_0^{\pi/2} \frac{1}{2} \left(\cos x - \cos 3x \right] \right)^2$$

$$= \frac{\cancel{8}2}{\pi^2} \left(\underbrace{\sin x \Big|_0^{\pi/2} - \frac{1}{3} \sin 3x \Big|_0^{\pi/2}}_{4/3} \right)^2$$

$$= \frac{32}{9\pi^2}$$

c) THE ENERGY IS PURELY KINETIC, SO WE JUST NEED TO MAKE SURE NOTHING FUNNY IS HAPPENING @ $x = a$ w/ THE DISCONTINUOUS 1^{st} DERIVATIVE.

WE NOTE IN GENERAL :

$$\int_a^b dx\, f f'' = \int_a^b \left(\underbrace{\cancel{\frac{d}{dx}(ff')}}_{d\,(ff')} - f'f'\,dx \right)$$

FOR US $f\ (\equiv \psi)$ VANISHES AT THE END POINTS, SO THE ENERGY IS THE SAME AS IT WAS WHEN THE WELL WAS ONLY a WIDE.

11.21

11.104] $\lambda \equiv \left[\omega^2 + \omega_1^2 - 2\omega\omega_1\cos\alpha \right]^{1/2}$

$= \omega_1 \left[1 + \left(\frac{\omega}{\omega_1}\right)^2 - 2\frac{\omega}{\omega_1}\cos\alpha \right]^{1/2}$

$\simeq \omega_1 \left[1 - 2\cos\alpha \, \omega/\omega_1 \right]^{1/2}$

$\simeq \omega_1 \left[1 - \frac{2}{2}\cos\alpha \frac{\omega}{\omega_1} \right]$

$= \omega_1 - \cos\alpha \, \omega$

SO FROM 11.105 WE FIND THE COEFFICIENT ~~TO~~ OF χ_+ TO BE

$\left(\cos\left[\lambda t/_2 \right] - i\sin\left[\lambda t/_2 \right] \right) e^{-i\omega t/_2}$

$= \cancel{e^{-i\lambda t/2}} \; e^{-i(\lambda+\omega)t/_2}$

$= e^{-i(\omega_1 + (1-\cos\alpha)\omega)t/_2}$

NOW $\theta(t) = -\frac{1}{\hbar}\left[\hbar\omega_1/_2 \right] t$

$= -\omega_1 t/_2$

~~SO~~ SUBTRACTING OFF THIS PHASE WE GET

$\gamma(t) = (\cos\alpha - 1)\,\omega t/_2$

SO THE BERRY PHASE IS $\gamma(T = \frac{2\pi}{\omega}) - \gamma(0)$

$= \pi \left(\cos\alpha - 1 \right)$

11.22 a) SINCE WE CAN ASSIGN WHATEVER PHASE WE LIKE TO THE EIGENFUNCTIONS $\psi(x;\alpha)$, i.e.

$$\psi(x;\alpha) = \exp(i\phi(\alpha))\,\beta e^{-\beta^2|x|}$$

$$\left\{\beta = \left(\frac{m\alpha}{\hbar^2}\right)^{1/2}\right\}$$

THE BERRY PHASE CAN ALWAYS BE MADE ZERO FOR A PATH ALONG SOME INTERVAL $[\alpha_1, \alpha_2]$.

THE ARGUMENT GOES:

- LET $\Psi(x,t)$ ~~(x)~~ BE ~~th~~ IN THE GROUND STATE @ ~~some~~ $t=0$, SO THAT

$$\Psi(x,0) = \exp(i\phi_0)\,\psi(x;\alpha(0))$$

FOR SOME $\phi_0 \in [0, 2\pi)$

- THE AD.AB. THM. THEN STATES THAT, @ SOME LARGE TIME t: ~~where~~ ~~$\alpha = \alpha_2$~~

$$\Psi(x,t) = \exp(i\phi_0)\exp(i\gamma(t)) \cdot \cancel{\exp(i\theta_0}\ \exp(i\theta(t)) \cdot \psi(x;\alpha(t))$$

- SUPPOSE FOR WHATEVER CHOICE WE MADE FOR $\phi(\alpha)$ WE GET SOME BERRY PHASE $\gamma(\alpha(t))$ / THE PHASE DEPENDS ONLY ON

THE TRAJECTORY $\alpha(t')$, $0 < t' < t$,
AND THUS FOR OUR CASE ON THE ENDPOINT
$\alpha(t)$]. WE CAN THEN MODIFY $\phi(\alpha)$
TO MAKE A NEW ~~ϕ~~ ONE i.e.

$$\phi(\alpha) \longrightarrow \phi'(\alpha) = \phi(\alpha) + \gamma(\alpha(t))$$

$$\psi(x;\alpha) \longrightarrow \psi'(x;\alpha) = \exp(+i\gamma(\alpha(t)))\psi(x;\alpha)$$

- THIS WILL NOT CHANGE $\theta(t)$, WHICH DEPENDS ONLY ON THE ENERGIES [AND NOT THE PHASES] OF THE $\psi(x;\alpha)$, SO WE WILL GET A NEW BERRY PHASE $\gamma'(\alpha(t)) = \gamma(\alpha(t)) - \gamma(\alpha(t)) = 0$

i.e.

$$\Psi(x,t) = e^{i\phi_0} e^{i\theta(t)} e^{i\gamma(\alpha)} \psi(x;\alpha)$$

$$= e^{i\phi_0} e^{i\theta(t)} e^{i\gamma(\alpha)} e^{-i\gamma(\alpha)} \psi(x;\alpha)$$

$$= e^{i\phi_0} e^{i\theta(t)} e^{i(0)} \psi'(x;\alpha)$$

i.e. $\gamma'(\alpha) = 0$

11.22

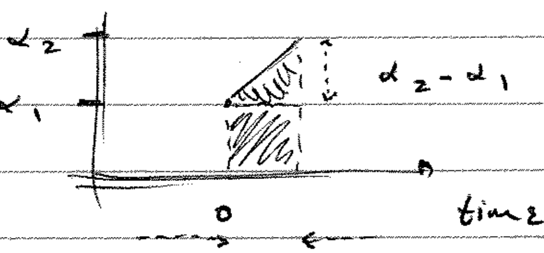
a) ~~GRIFFITHS STATES THAT α INCREASES, ONLY SO THAT $\psi(x;\alpha_1) \neq \psi(x;\alpha_2)$. WE ARE THEREFORE FREE TO ASSIGN WHATEVER PHASE ~~DIFF~~ TO ~~EITHER OF THE~~ THE LOWER CASE ψ's IN EQN 11.93 TO MAKE THE GEOMETRIC PHASE VANISH.~~

~~SEE BELOW b)~~

b) $$\theta(t) = \frac{-1}{\hbar}\int_0^t dt'\, E(t')$$

$$= \frac{-1}{\hbar}\int_0^t dt' \left[- \frac{m[\alpha_1 + ct']}{\hbar^2} \right]$$

$$= m\hbar^{-3}\int_0^t dt' \left[\alpha_1 + ct'\right]$$



$$\frac{[\alpha_2 - \alpha_1]}{c}$$

$$= \frac{m\hbar^{-3}}{c}\left[\alpha_1 [\alpha_2 - \alpha_1] + \frac{1}{2}[\alpha_2 - \alpha_1]^2 \right]$$