11.13

THE FINAL STATE IS THE ONE $n=1$ STATE:

$$n=1,\ \ell=0,\ m_\ell=0$$

FROM EQ^N 11.75, WE THEN FIND THAT THE LIFETIME OF THE $n=2$, $\ell=0$ STATE IS INFINITE [TO 1ST ORDER IN P.T.].

FOR THE OTHER STATES [$n=2,\ \ell=1,\ m=0,\pm1$] WE ARE ASKED TO EVALUATE THE MATRIX ELEMENTS

$$|\vec{\wp}|^2 \propto |\langle 100|\vec{r}|21m\rangle|^2$$

$$= \sum_{q=\{-1,0,+1\}} |\langle 100|r_q|21m\rangle|^2$$

[WHERE THE r_q ARE DEFINED @ THE BEGINNING OF SEC. 6.7.2]

$$= \sum_q \left| C^{1\ 1\ 0}_{m\ q\ 0} \langle 10\|r\|21\rangle \right|^2$$

[WHERE $C^{j_1 j_2 J}_{m_1 m_2 M}$ IS A CLEBSCH GORDAN COEFF. & $\langle 10\|r\|21\rangle$ IS THE RED. MAT. EL. DISCUSSED IN SEC. 6.7]

$$= \sum_q \left| \delta_{11}\, \delta_{m-q}\, \frac{(-1)^{1-m}}{\sqrt{3}} \langle 10 \| r \| 21 \rangle \right|^2$$

$$= \frac{1}{3} \left| \langle 10 \| r \| 21 \rangle \right|^2$$

SO THAT $|\vec{\wp}|^2$ IS INDEPENDENT OF m + SO IT SUFFICES TO CALCULATE IT FOR ONE STATE.

$m = 0$ WILL BE THE EASIEST.

$$\langle 100 | \vec{r} | 210 \rangle = \langle 100 | r_0 \hat{z} | 210 \rangle$$

AND SO $|\langle 100 | \vec{r} | 210 \rangle|^2 = |\langle 100 | r_0 | 210 \rangle|^2$, FOR REASONS IDENTICAL TO THE STEPS ABOVE.

FOR $\langle 100 | r_0 | 210 \rangle$

$$= \langle 100 | r \cos\theta | 210 \rangle,$$

- THE PHI INTEGRAL GIVES 2π
- FOR THETA: WE GET $1/\sqrt{3}$ (SEE STARK EFFECT HW PROBLEM)
- FOR r: $\int_0^\infty dr\, r^2 R_{10}(r)\, r\, R_{21}(r)$

$$= a_0 \int_0^\infty dx\, x^3 \left[\frac{1}{6}\right]^{1/2} x\, e^{-3/2\, x}$$

$$= a_0 / \sqrt{6} \left[\frac{3}{2}\right]^5 \int_0^\infty dx\, x^4 e^{-x} = \frac{a_0}{\sqrt{6}} \cdot \frac{3^6}{4}$$

THE REST IS JUST PLUGGING INTO EQUATION 11.63

11.16

FROM SELECTION RULES, $n = 3$, $\ell = 0$, $m = 0$ CAN ONLY DECAY TO THE

$n = 2$, $\ell = 1$, $m = -1, 0, +1$

STATES. FROM THERE THESE STATES MAY DECAY TO THE G.S. $n = 0$, $\ell = 0$, $m = 0$. THIS PROBLEM IS ESSENTIALLY 11.13 IN REVERSE, SO THAT THE INITIAL $n = 3$ STATE DECAY W/ EQUAL PROB TO EACH OF THE $n = 2$, $\ell = 1$ STATES.

THE LIFETIME CAN BE SEEN TO BE IDENTICAL TO THE ONES CALCULATED ~~FROM~~ IN 11.13, EXCEPT FOR:

- THE RADIAL INTEGRALS ARE ~~WILL BE~~ DIFFERENT
- A FACTOR OF 3, SINCE THE STATE CAN DECAY TO THREE DISTINCT STATES.

11.24

a) ~~LET $|\psi\rangle$ BE A STATE,~~ ~~[illegible]~~
~~[illegible]~~ AND SUPPOSE @ A TIME t ~~[illegible]~~ ITS AMPLITUDE ALONG THE n^{TH} EIGENSTATE OF H_0 IS $b_n(t)$, i.e.

$$\langle n|\psi\rangle(t) \equiv b_n(t)$$

EXTRACT THEN A PHASE FACTOR $e^{-iE_nt/\hbar}$ FROM EACH $b_n(t)$, i.e. DEFINE

$$C_n(t) \equiv b_n(t)\exp(+iE_nt/\hbar)$$

THEN

$$|\psi\rangle(t) = \sum_n |n\rangle\langle n|\psi\rangle(t)$$

$$= \sum_n c_n e^{-iE_nt/\hbar}|n\rangle$$

~~FROM THE TDSE WE THEN GET~~

$$\cancel{\dot{C}_m(t) = \langle m| \sum_n \left(\partial_t\right.}$$

THEN:

$$\partial_t|\psi\rangle = \sum_n\left[\dot{C}_n - iE_n/\hbar\, C_n\right]e^{-iE_nt/\hbar}|n\rangle$$

OR $$\langle m|\partial_t|\psi\rangle = \left[\dot{C}_m - iE_m/\hbar\, C_m\right]e^{-iE_mt/\hbar}$$

[TDSE]

But $\partial_t |\psi\rangle = \frac{H|\psi\rangle}{i\hbar} = \frac{-i}{\hbar}[H_0 + H']|\psi\rangle$

$= \frac{-i}{\hbar} \sum_n [E_n + H'] c_n e^{-iE_n t/\hbar} |n\rangle$

or $\langle m|\partial_t|\psi\rangle = \frac{-i}{\hbar} E_m c_m e^{-iE_m t/\hbar} - \frac{i}{\hbar}\sum_n V_{mn} c_n e^{-iE_n t/\hbar}$

Comparing the two expressions we find:

$\dot{c}_m e^{-iE_m t/\hbar} = \frac{-i}{\hbar} \sum_n V_{mn} e^{-iE_n t/\hbar} c_n$

or:

$\dot{c}_m = \frac{-i}{\hbar} \sum_n c_n V_{mn} e^{i[E_m - E_n]t/\hbar}$

b) Expanding $c_n(t)$ in a power series in H' and matching coeffs we find:

$\dot{c}_m^{(1)} = \frac{-i}{\hbar} \sum_n c_n^{(0)} V_{mn} e^{i[E_m - E_n]t/\hbar}$

$= \frac{-i}{\hbar} \sum_n \delta_{nN} V_{mn} e^{i[E_m - E_n]t/\hbar}$

so $c_m(t) \simeq c_m^{(0)} + c_m^{(1)}$

$= \delta_{mN} - \frac{i}{\hbar} \int_0^t dt' \, V_{mN}(t') e^{i[E_m - E_N]t'/\hbar}$

a),
11.37 b)

EACH MEASUREMENT IS PROBABILISTICALLY INDEPENDENT, SO THE PROBABILITY OF n ~~CONSECUTIVE~~ CONSECUTIVE "NOT DECAYED" MEASURED IS THE PRODUCT OF ~~THE~~ EACH:

$$\text{PROB} = \left(1 - \alpha\left(t/n\right)^2\right)^n$$

$$\doteq 1 - n\alpha\left(t/n\right)^2$$

$$= 1 - \alpha\left(\sqrt{n}\, t/n\right)^2$$

$$= 1 - \alpha\left(t/\sqrt{n}\right)^2$$

SO THAT TAKING n OBSERVATIONS OVER A TIME t IS THE SAME AS MAKING <u>ONE</u> OBSERVATION AFTER A TIME $t/\sqrt{n}$.