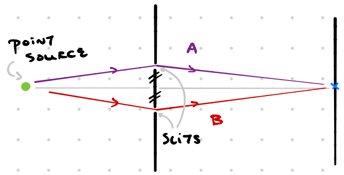
LECTURE 26 QUESTIONS

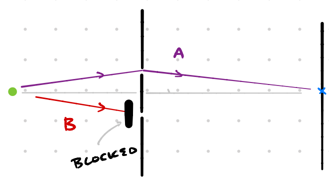
Q1 Two-Slit Interference



In the following we consider the point (×) midway between the two slits.

a) What is the path-length difference $\Delta\Gamma$ for the two possible paths A + B that light can take to reach (×)?

b) Suppose we block the lower slit, eliminating path B: and measure the intensity at (×) to be $I_A^{\times}$.

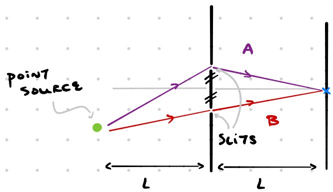


- What intensity $I_{A+B}^{\times}$ do we expect to measure when we unblock the slit?

[Hint: Amplitudes add]

c) If the power absorbed @ (×) is 4× (i.e. > 2×) the power absorbed w/ one slit blocked [i.e. $I_{A+B}^{\times} = 4\, I_A^{\times}$ (pt. (b))], where is this extra energy "coming from"? How is energy conserved?

d) Now suppose we move our point source down, as illustrated:



- is Δr still zero?

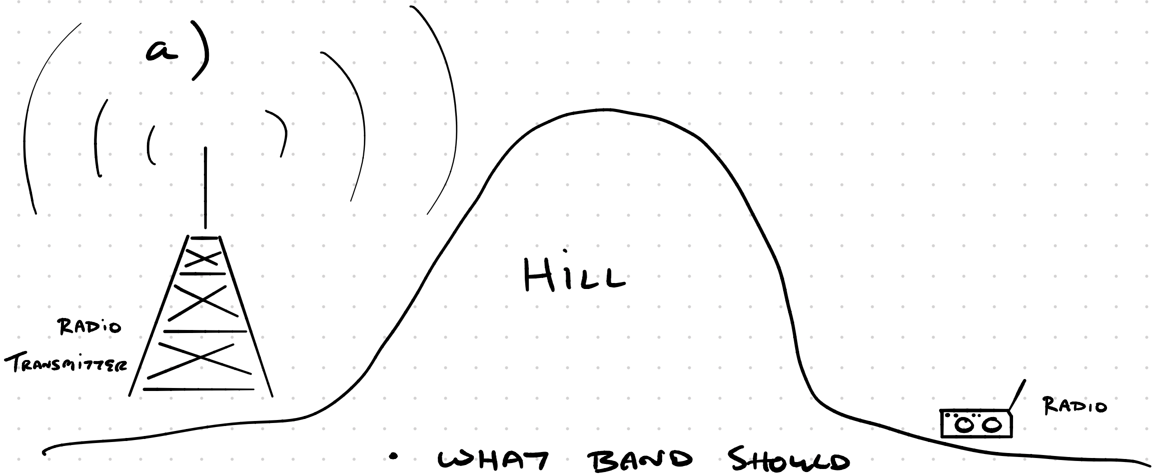
e) Where do we need to move (×) to so that Δr is zero again?

Suppose for simplicity that the slits are halfway between the screen & the point source.

In general, what does shifting the point source do to the interference pattern @ the screen?

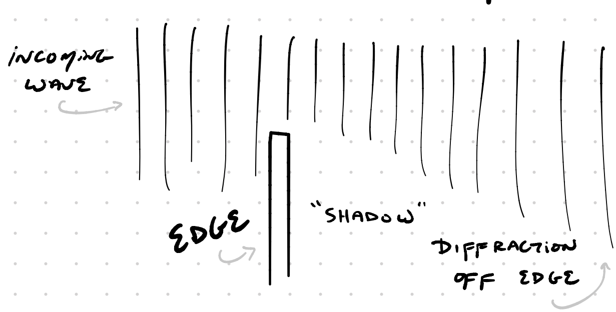
Q2 Diffraction

a)

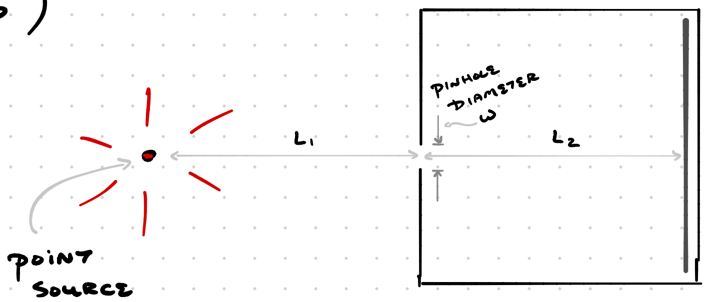


- What band should the transmitter broadcast on to optimize reception @ the radio receiver on the other side of the hill:
  - FM [$f \simeq 100$ MHz] or
  - AM [$f \simeq 1$ MHz] ?

[Hint: waves diffract past edges in a manner similar to diffraction past slits]



b)



- IN RAY OPTICS, THE SPOT FORMED BY A POINT SOURCE ON THE SCREEN OF A PINHOLE CAMERA GETS SMALLER AS THE PINHOLE DIAMETER DECREASES SINCE FEWER RAYS ARE ABLE TO MAKE IT INTO THE CAMERA.

- WAVE OPTICS, HOWEVER, WARNS US THAT WHEN THE PINHOLE GETS TOO SMALL, DIFFRACTION WILL ACT TO INCREASE THE SPOT SIZE.

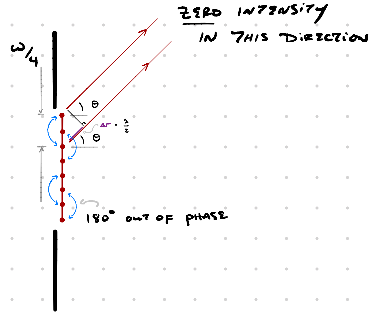
- (ROUGHLY)* HOW LARGE SHOULD YOU MAKE THE PINHOLE GET THE SMALLEST SPOT SIZE ON THE CAMERA SCREEN?

  I.E. WHEN DO THE RAY & WAVE THEORIES PREDICT THE SAME SPOT SIZE?

* I.E. USE THE EQUATION $\frac{2L\lambda}{w}$ FOR THE DIFFRACTION SPOT SIZE, EVEN THOUGH, STRICTLY SPEAKING, THIS EQ^N^ APPLIES FOR SLIT-SHAPED APERTURES, NOT CIRCULAR.

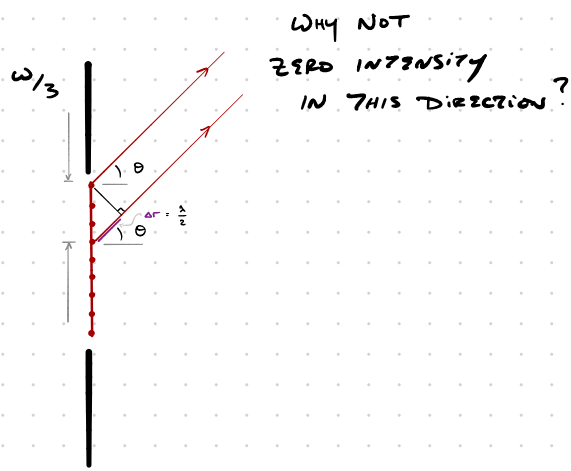
c) IN THE NOTES WE ARGUED THAT DESTRUCTIVE INTERFERENCE BETWEEN POINTS SEPARATED BY AN EVEN DIVISION OF THE SLIT WIDTH $\left[d = \frac{w}{2}, \frac{w}{4}, \frac{w}{6}, \ldots \right]$ RESULTED IN TOTAL DESTRUCTIVE INTERFERENCE @ THE SCREEN:

E.G. $d = \frac{w}{4}$:



• WHY DO WE NOT OBSERVE ZERO INTENSITY @ AN ANGLE θ WHERE WE HAVE DESTRUCTIVE INTERFERENCE BETWEEN POINTS SEPARATED BY AN ODD DIVISION OF THE SLIT WIDTH $\left[d = \frac{w}{1}, \frac{w}{3}, \frac{w}{5}, \ldots \right]$?

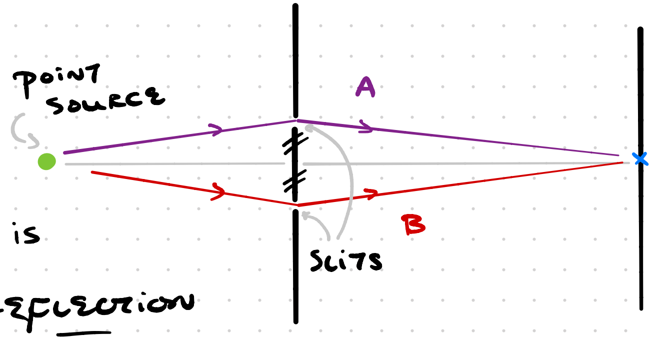
E.G. $d = \frac{w}{3}$:



ANSWERS

Q1

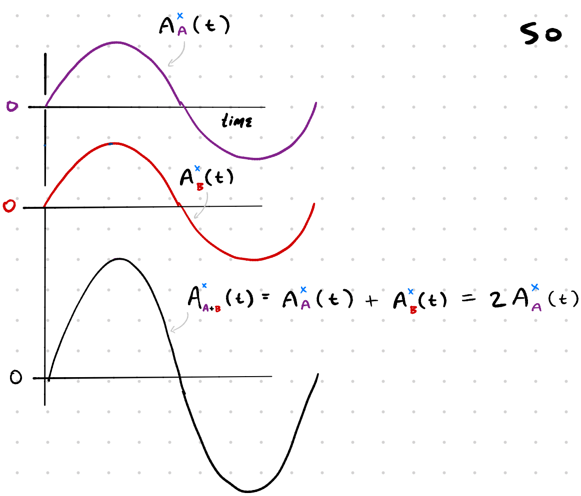
a)



PATH B is just a reflection of path A. Distances traveled are equal so $\Delta r = 0$.

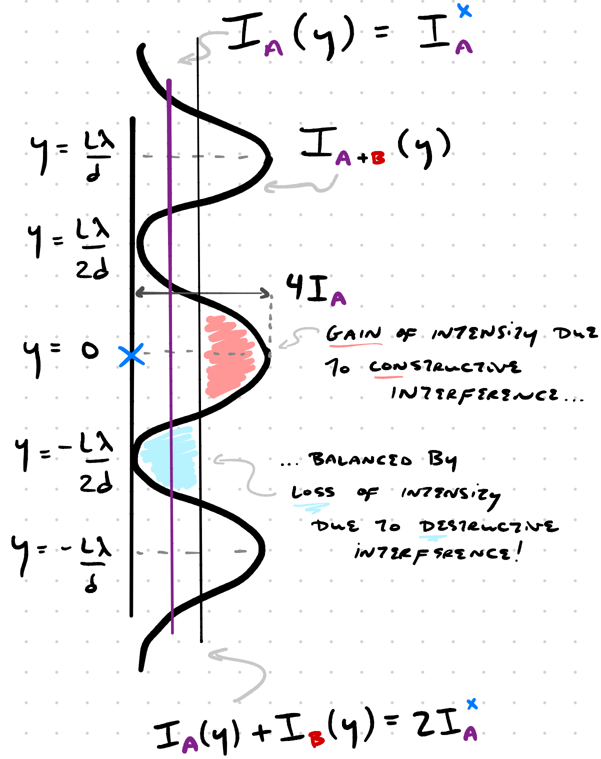
We expect constructive interference here.

b) The amplitudes will add, doubling the amplitude @ (×) since they add "in phase" $(\Delta r = 0)$:

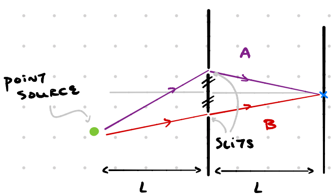


so
$$I_{A+B}^{\times} = \left(A_{A+B}^{\times}\right)^2$$
$$= \left(A_A^{\times} + A_B^{\times}\right)^2$$
$$= \left(2A_A^{\times}\right)^2$$
$$= 4\left(A_A^{\times}\right)^2$$
$$= 4 I_A^{\times} \checkmark$$

c) Pattern on screen:

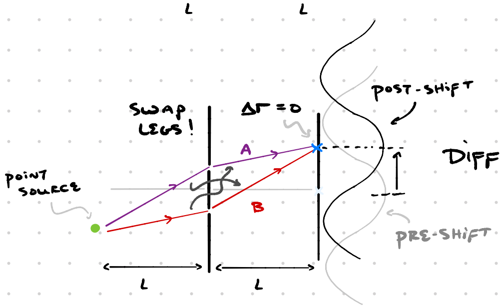


d)



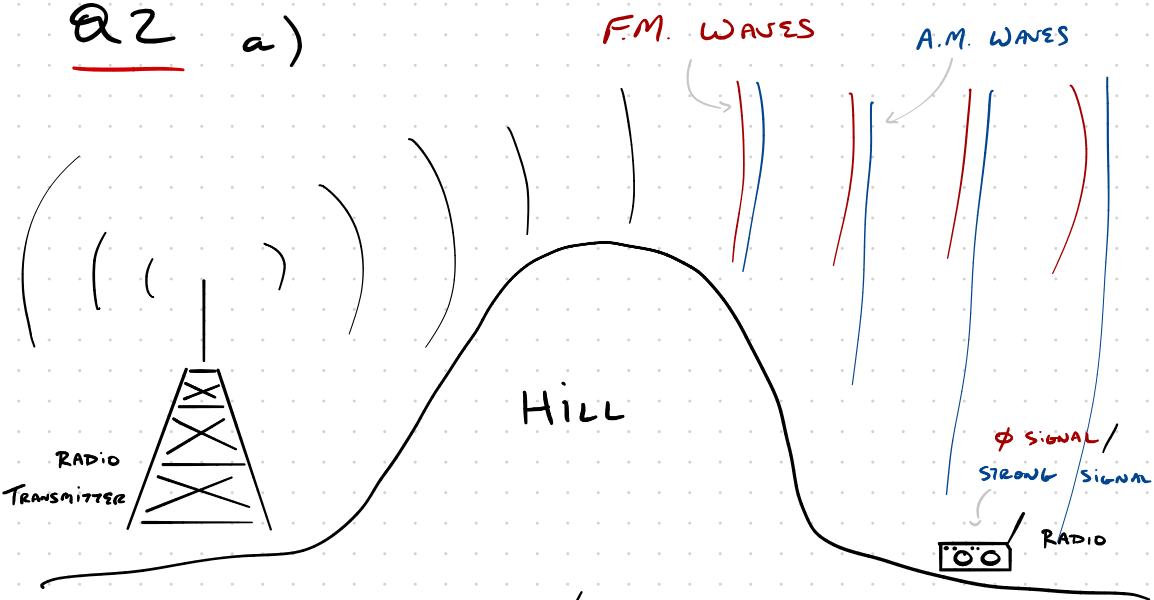
No! Path A clearly longer than path B.

e)



Diffraction pattern shifts upwards.

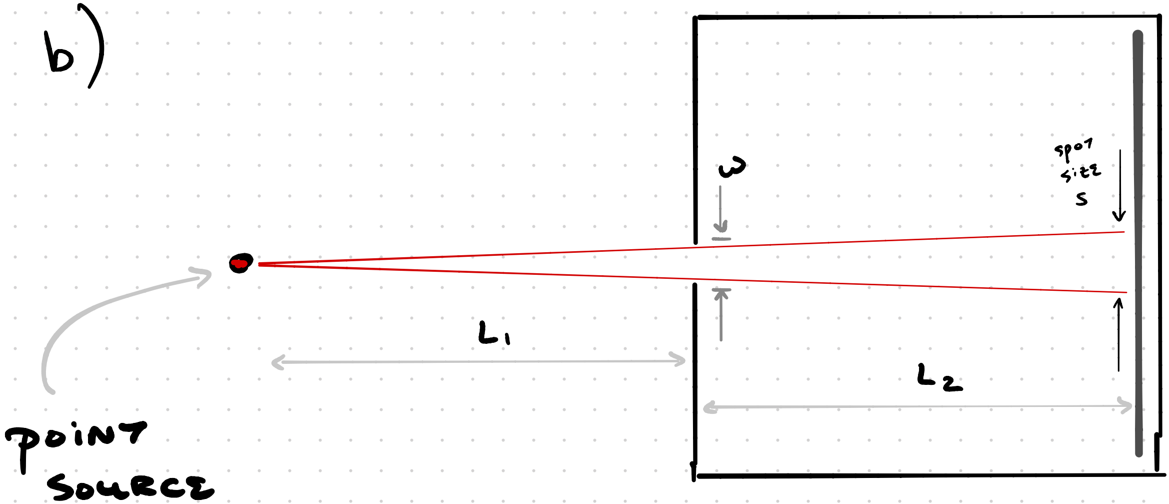
Q2 a)



WEAK / STRONG
DIFFRACTION:
BIG / SMALL SHADOW

- DIFFRACTION EFFECTS STRONGER @ LONG WAVELENGTHS ~ LOW FREQUENCIES
  [$f\lambda = c$]
- BETTER RECEPTION @ A.M. [$\simeq 1$ MHz] vs. F.M. [$\simeq 100$ MHz]

b)



Spot size (ray optics):

[similar triangles]

$$\frac{S_{ray}}{L_1+L_2} = \frac{w}{L_1} \longrightarrow S_{ray} = \frac{L_1+L_2}{L_1} w$$

Spot size (wave optics):

$$S_{wave} \simeq \frac{2 L_2 \lambda}{w}$$ ← from notes

Min. spot size when $S_{ray} = S_{wave}$

$$\frac{L_1+L_2}{L_1} w = \frac{2 L_2 \lambda}{w}$$

$$\longrightarrow w^2 = \frac{2 L_1 L_2 \lambda}{L_1 + L_2}$$

$$w = \left[\frac{2 L_1 L_2 \lambda}{L_1 + L_2} \right]^{1/2}$$

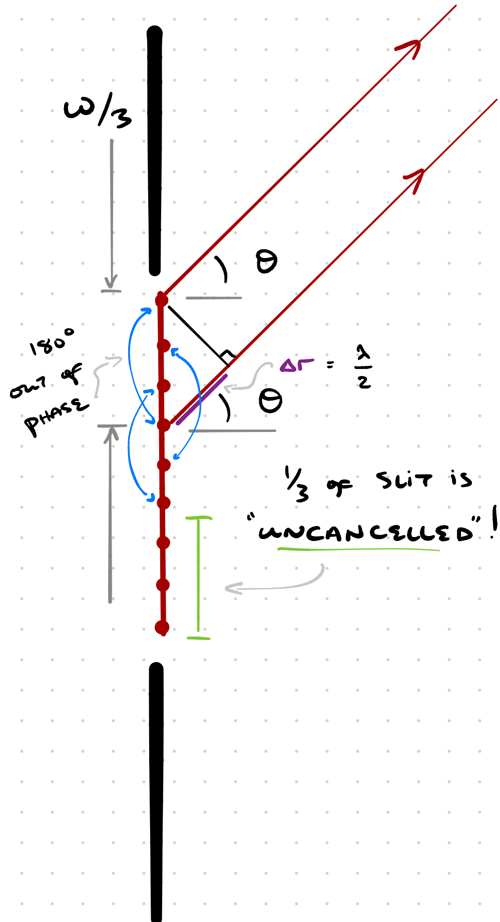
E.G. for $L_1 = L_2 = 1m$ + $\lambda = 1 \mu m$ (≃ red light)

$\longrightarrow w \simeq 1mm \simeq$ human eye pupil diameter

hmm.

c)

• PAIRS • JOINED BY ⟷ CANCEL EACH OTHER, BUT BOTTOM THIRD OF SLIT HAS NO POINTS TO "PAIR OFF" WITH!



• DESTRUCTIVE INTERFERENCE IS THEREFORE NOT TOTAL, I.E. INTENSITY IS NON-ZERO.

• FOR LARGE ODD DIVISIONS [E.G. $d = \frac{w}{101}, \frac{w}{1001}$], ONLY TINY FRACTION OF SLIT WIDTH IS UNCANCELLED [$\frac{1}{101}, \frac{1}{1001}$], SO INTENSITY, WHILE STRICTLY SPEAKING IS NON-ZERO, IS VERY LOW.

• THIS IMPLIES THAT INTENSITY OF SINGLE SLIT DIFFRACTION PATTERN IS VERY DIM @ HIGH ANGLES, SINCE ANY HIGH ANGLE IS CLOSE TO AN ANGLE WHERE $\Delta r = \pm \frac{\lambda}{2}$ FOR SOME LARGE DIVISION [EVEN OR ODD!] OF w.