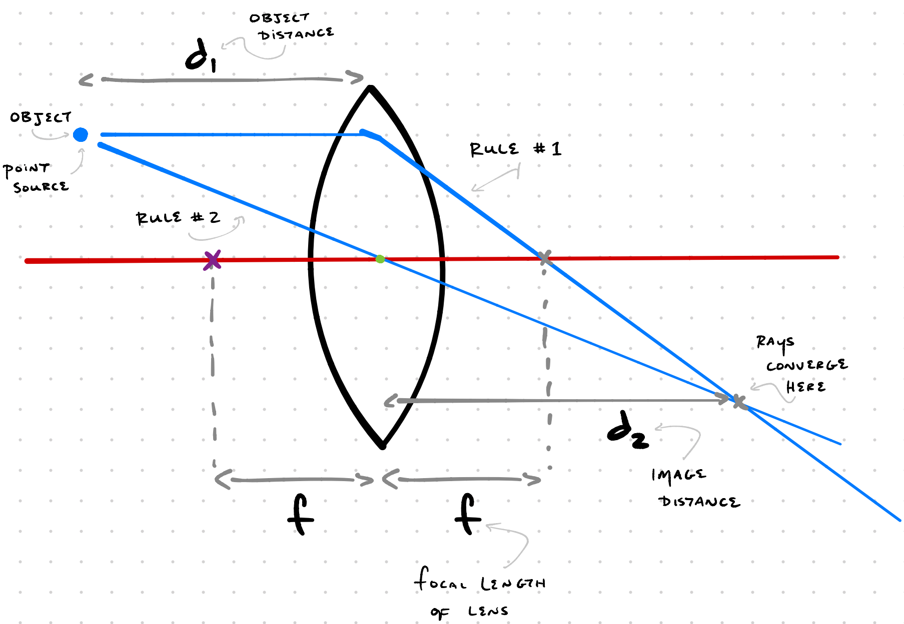
LECTURE 25 QUESTIONS

Q1 LENSES

a) IN THE NOTES WE ILLUSTRATED HOW RAYS LEAVING A POINT SOURCE A DISTANCE d_1 FROM THE LENS CONVERGE AT A POINT A DISTANCE d_2 FROM THE LENS, WHERE:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$



- HOWEVER, WE ONLY SHOWED THIS FOR TWO RAYS THAT REFRACT THRU THE LENS. DO THE OTHER RAYS CONVERGE @ d_2 AS WELL*? SHOW THAT THERE IS AT LEAST A THIRD RAY INTERSECTING @ d_2 BY DRAWING A RAY LEAVING THE OBJECT AND CROSSING THRU THE FOCAL POINT ON THE OBJECT SIDE OF THE LENS. APPLYING RULE #1 "IN REVERSE". IN WHAT DIRECTION DOES THIS RAY EXIT THE LENS?

* TO A GOOD APPROX. THEY DO. DEVIATIONS FROM PERFECT CONVERGENCE ARE CALLED "ABBERATIONS".

b) WHAT DOES THE THIN LENS EQ^N

$$\boxed{\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}}$$

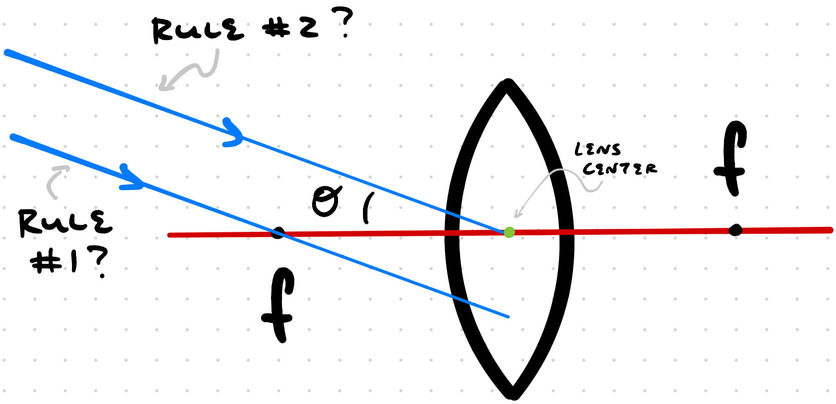
PREDICT FOR THE IMAGE DISTANCE d_2 OF AN OBJECT LOCATED FAR AWAY FROM THE LENS [I.E. $d_1 \gg f$]?

IN LIGHT OF RULE #1, WHAT DOES THIS TELL YOU ABOUT THE RELATIVE DIRECTION OF THE RAYS FROM A DISTANT POINT SOURCE STRIKING A LENS?

* AKA "COLLIMATED" LIGHT

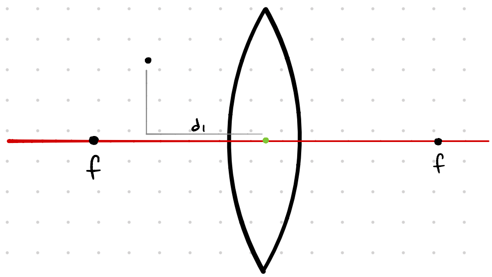
c) SHOW THAT PARALLEL RAYS* THAT ARE'NT PARALLEL TO THE LENS' SYMMETRY AXIS STILL CONVERGE A DISTANCE f AWAY FROM THE LENS :

i.e. $\theta \neq 0$



- HOW FAR BELOW THE FOCAL POINT DO THE RAYS CONVERGE ?
- IS YOUR ANSWER CONSISTENT W/ THE THIN LENS EQUATION & THE RELATION $\frac{h_2}{h_1} = -\frac{d_2}{d_1}$?

d) WHY IS d_2 NEGATIVE IF $d_1 < f$?



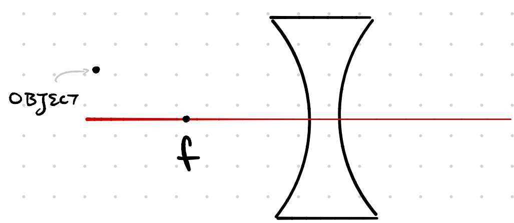
- DRAW TWO RAYS LEAVING THE OBJECT, ONE DEMONSTRATING RULE #1 & THE OTHER DEMONSTRATING RULE #2. WHERE DO THESE RAYS APPEAR TO CONVERGE?

e) SHOW THAT THE IMAGE DISTANCE d_2 FOR A DIVERGING LENS IS ALWAYS NEGATIVE.

- SHOW THIS TWO WAYS, USING

  1: THE THIN LENS EQUATION &

  2: RAY TRACING [i.e. RULES #1 & #2].



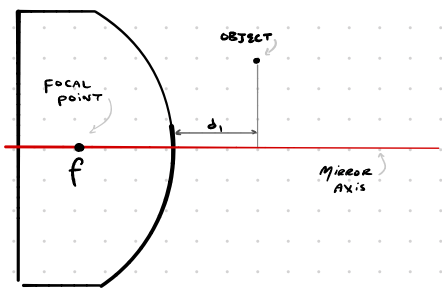
Q2 Mirrors

* See the answer to Q1(d) for definition of these terms.

Show, using ray tracing [i.e. Rules #1* + #2*] that:

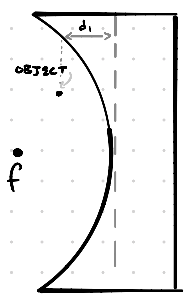
* See next page for description

a) The image of any object ($d_1 > f$ or $d_1 < f$) produced by a convex mirror is is upright + virtual*.



Convex Mirror

b)



Concave Mirror

- The image of an object located inside the focal point of a mirror (i.e. $d_1 < f$) is also upright and virtual*.
- To avoid confusion, draw only two rays [i.e. do not also apply Rule #1 "in reverse" as shown in the part (a) answer]

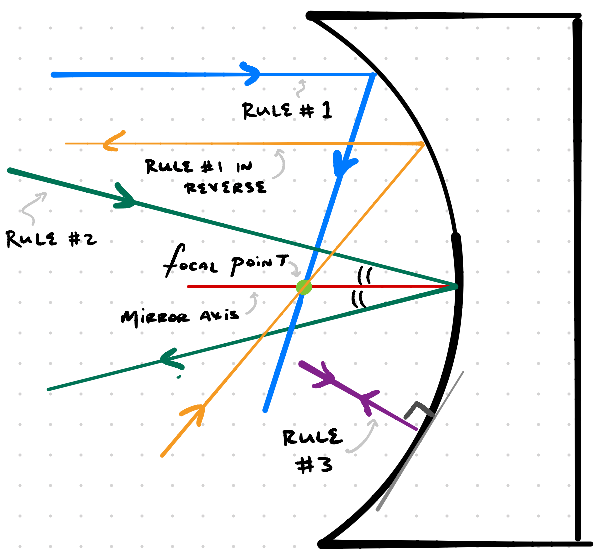
Ray Tracing Rules for Mirrors:

Rule #1: Rays parallel to the mirror axis intersect the mirror axis at the focal point of the mirror.*

* For convex mirrors, the rays only appear to intersect the focal point, which lies on the other side of the mirror.

Rule #2: Rays hitting the mirror @ the mirror axis exit in a direction so that the two angles made w/ the mirror axis are the same.

Rule #3*: Rays ⊥ to a tangent retrace their path.

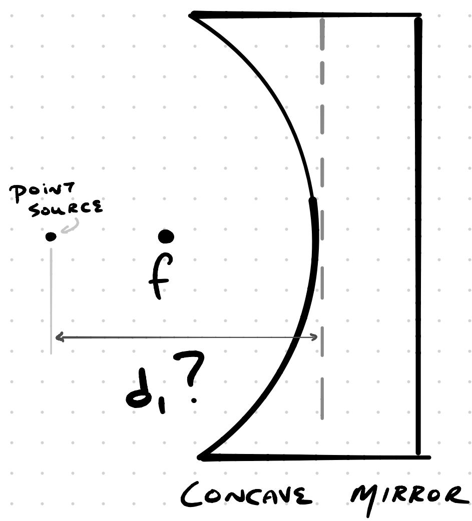


Concave Mirror

* I don't use this rule in the notes or these questions but include here for completeness.

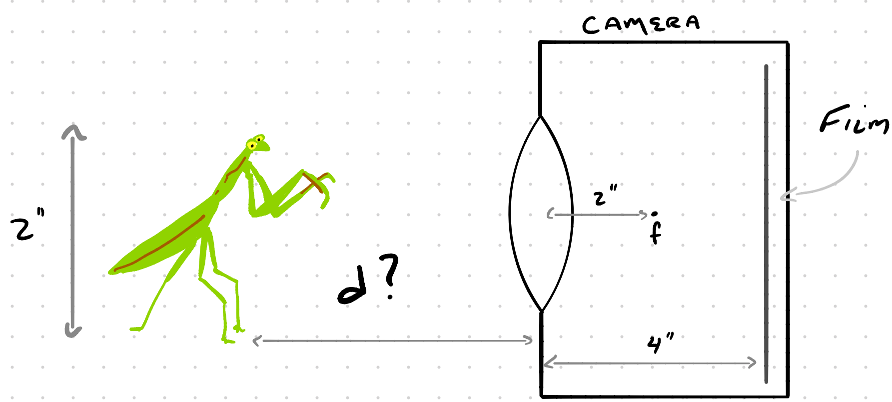
c)

WHERE WOULD YOU WANT TO PLACE A POINT SOURCE SO THAT ITS REFLECTED RAYS ARE "COLLIMATED" [i.e. PARALLEL]?



Q3 Imaging Systems

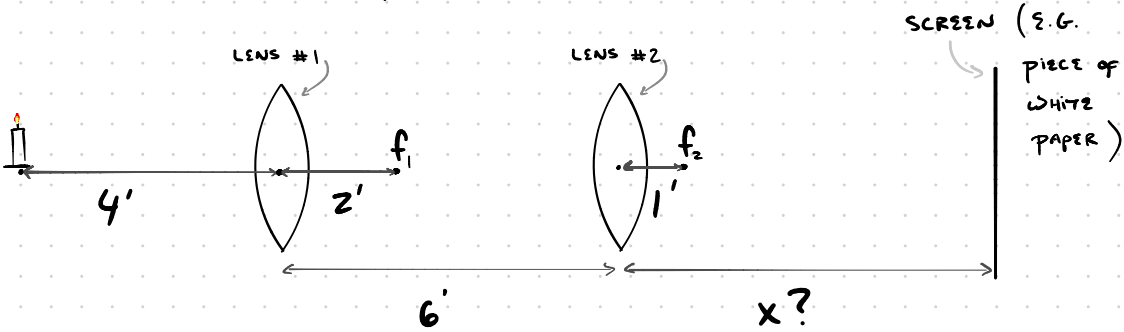
a) Suppose you want to take a picture of a bug using your camera which consists of a +2" focal length lens with film located 4" behind the lens. How close should you get the camera to the bug to get a sharp image?



- If the bug is 2" tall, how big should the film be to capture the entire bug?

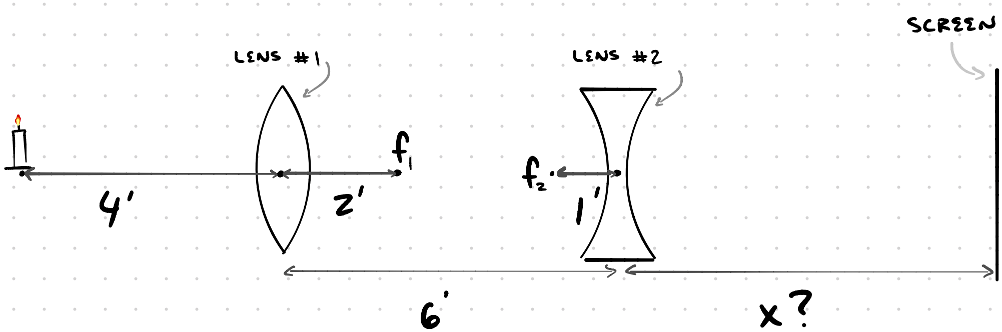
b) WHERE SHOULD A SCREEN BE PLACED IN ORDER TO OBSERVE A SHARP IMAGE OF A CANDLE FORMED BY THE SECOND LENS?

- HINT: APPLY THIN LENS EQ^N TWICE.

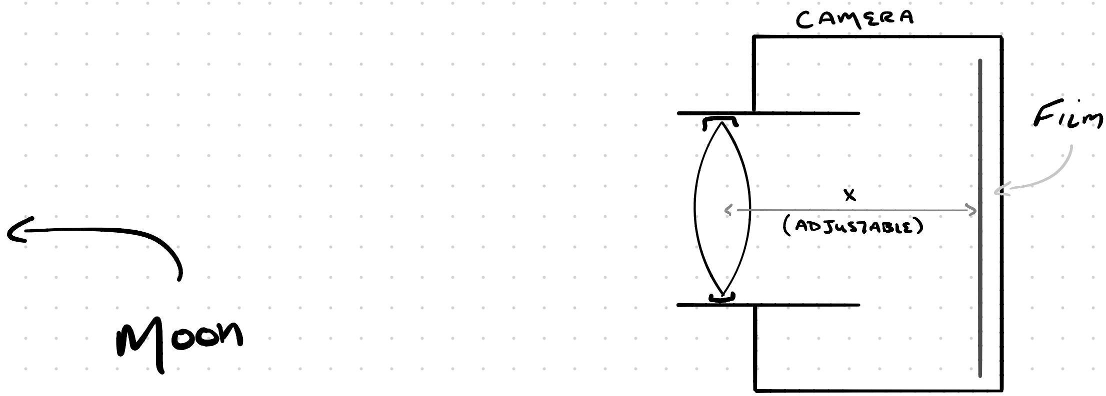


- IS THE IMAGE UPRIGHT OR INVERTED?

- (TRICK QUESTION) HOW DOES YOUR ANSWER CHANGE IF WE REPLACE THE SECOND LENS W/ A DIVERGING LENS?



c) ANOTHER CAMERA HAS A LENS w/ A +5" FOCAL LENGTH & AN "ADJUSTABLE FOCUS":



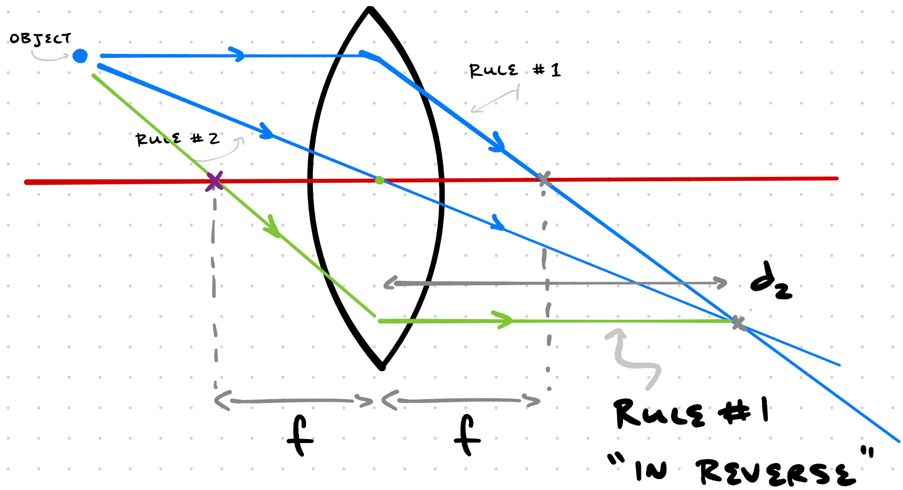
- TO WHAT DEPTH SHOULD THE LENS - FILM DISTANCE (x) BE ADJUSTED IN ORDER TO CAPTURE AN IMAGE OF THE MOON?

- IF THE MOON SUBTENDS AN ANGLE OF ABOUT 1° IN THE SKY, HOW BIG OF A SPOT DOES IT MAKE ON THE FILM [WHEN IN FOCUS]?

ANSWERS

Q1

a)

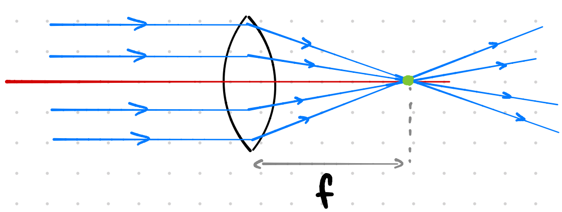


b) $$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f} \longrightarrow \frac{1}{d_2} = \frac{1}{f} - \cancel{\frac{1}{d_1}}^{\,\to 0}$$

$(d_1 \gg f)$

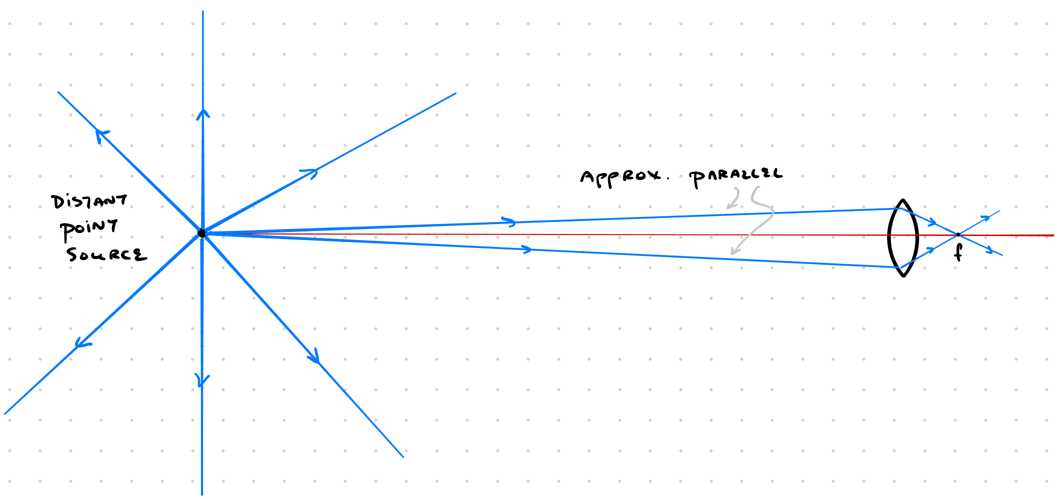
$$\longrightarrow \quad d_2 \simeq f \quad \text{if} \quad d_1 \gg f$$

- RULE #1 STATES THAT RAYS PARALLEL TO THE LENS' SYMMETRY AXIS CROSS THE AXIS @ AT THE FOCAL POINT OF THE LENS:



ANSWER TO Q1(b) CONT.

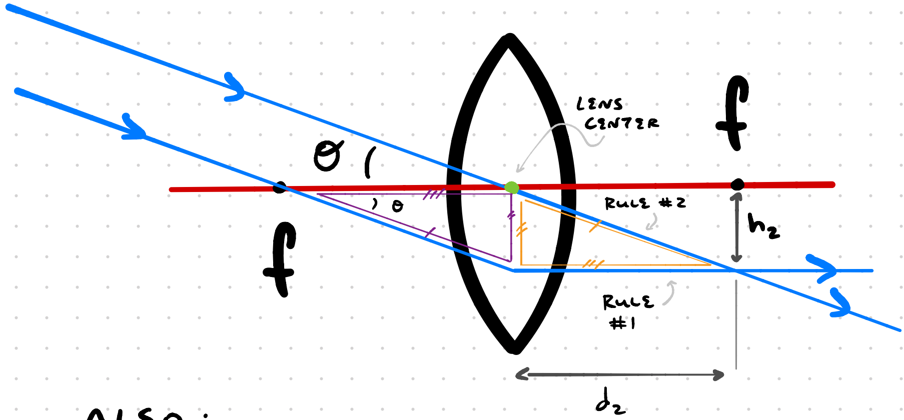
- IF THIS IS ALSO THE DISTANCE AT WHICH LIGHT FROM A DISTANT OBJECT CONVERGES, THEN THE RAYS FROM THE OBJECT THAT STRIKE THE LENS ARE APPROXIMATELY PARALLEL:



c) Purple + orange triangles are congruent:

Since $= f$

Then $d_2 = \quad = f$

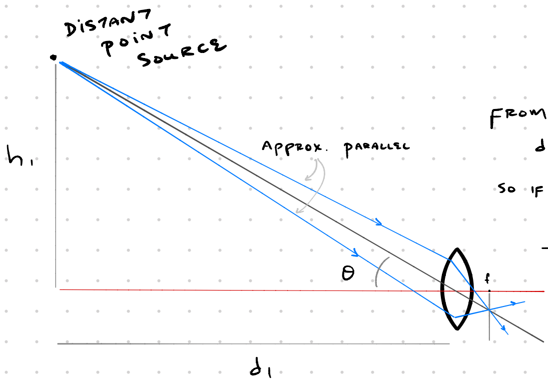


ALSO:

$=$

So since $= \quad \times \tan\theta = f\tan\theta$

Then $|h_2| = \quad = f\tan\theta$



via thin lens eq^n

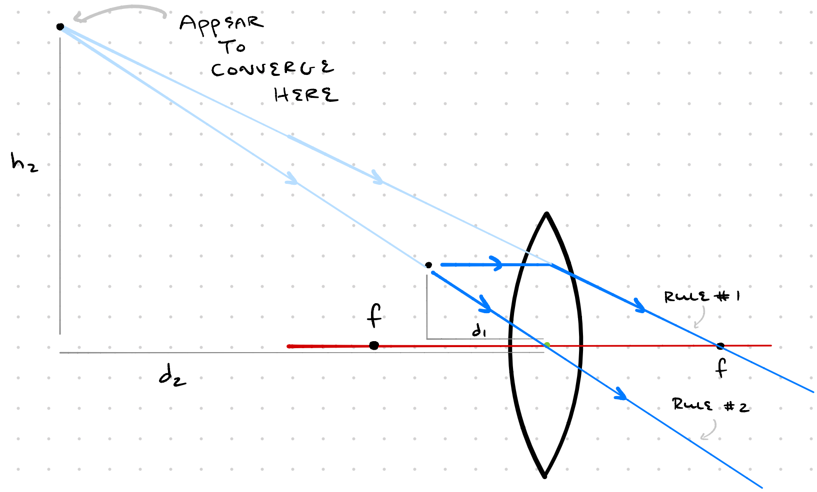
From (b):

$d_2 \approx f$ ✓

So if $\frac{h_2}{h_1} = -\frac{d_2}{d_1}$:

$\longrightarrow h_2 = -d_2 \times \frac{h_1}{d_1} = -f \times \tan\theta$ ✓

(h_2 negative since object + image on opposite sides of lens axis) d)



d_2 NEGATIVE: IMAGE ON SAME SIDE OF LENS AS OBJECT.

- IF $d_2 < 0 \longrightarrow h_2 = -d_2 \times \dfrac{h_1}{d_1} > 0$, SO:

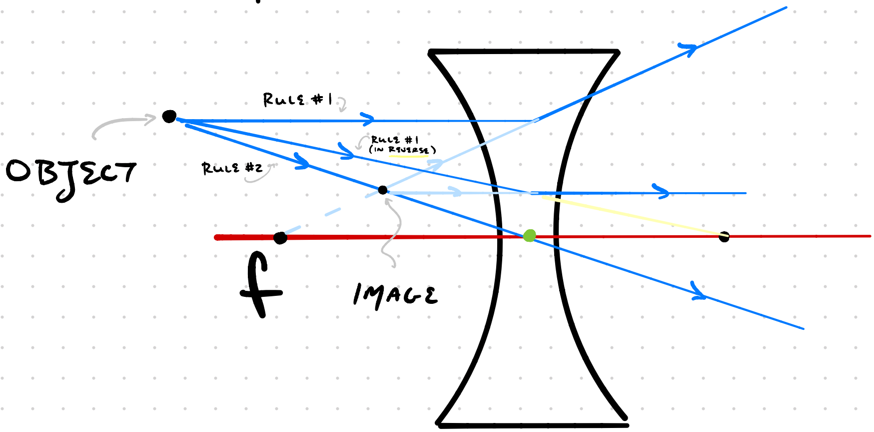
h_2 POSITIVE: IMAGE IS ON SAME SIDE OF LENS AXIS AS OBJECT.

- TERMINOLOGY:
  - WHEN $\left\{ \begin{array}{l} d_2 \text{ is positive} \\ h_2 \text{ is negative} \end{array} \right.$, WE SAY THE IMAGE IS INVERTED, BECAUSE THE IMAGE AND OBJECT ARE ON OPPOSITE SIDES OF THE LENS AXIS. WE ALSO SAY IN THIS CASE THAT THE IMAGE IS REAL, SINCE RAYS ACTUALLY CONVERGE @ THIS POINT.*
  - SIMILARLY, WHEN $\left\{ \begin{array}{l} d_2 \text{ is negative} \\ h_2 \text{ is positive} \end{array} \right.$, WE SAY THE IMAGE IS UPRIGHT OR "NOT INVERTED" SINCE OBJECT & IMAGE ARE ON SAME SIDE OF AXIS. IN THIS CASE THE IMAGE IS VIRTUAL, SINCE RAYS ONLY APPEAR TO CONVERGE @ THE IMAGE LOCATION.*

* TRUE FOR IMAGES FORMED BY A SINGLE LENS/MIRROR.

e)

RAY TRACING:



- IMAGE ON SAME SIDE OF LENS AS OBJECT.

  $\longrightarrow$ d_2 NEGATIVE. ✓

- FROM THIN LENS EQUATION:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

DOESN'T MATTER if $d_1 > f$ or $d_1 < f$

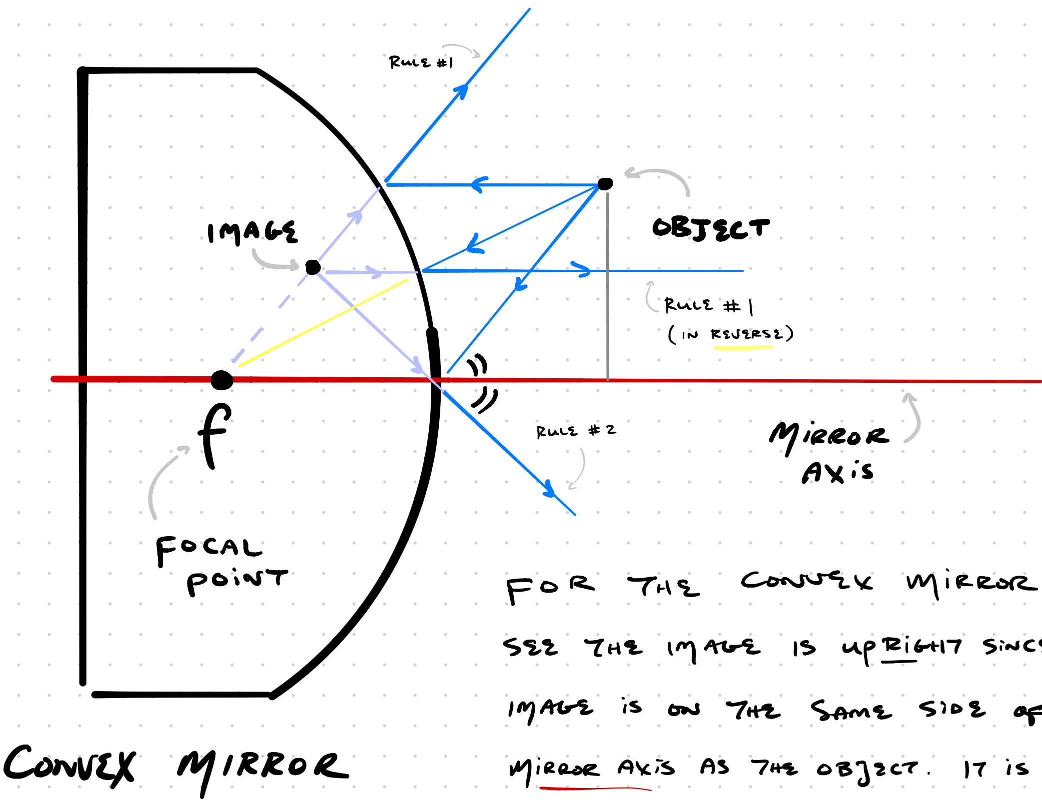
$$\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} < 0 \longrightarrow d_2 < 0 \checkmark$$

NEGATIVE FOR DIVERGING LENS

POSITIVE BY CONVENTION

- SO DIVERGING LENS ALWAYS FORMS UPRIGHT, VIRTUAL IMAGE.

Q2 a)



For the convex mirror we see the image is upright since the image is on the same side of the mirror axis as the object. It is also virtual since rays only appear to converge @ image location. ✓

Q2 (a) cont.

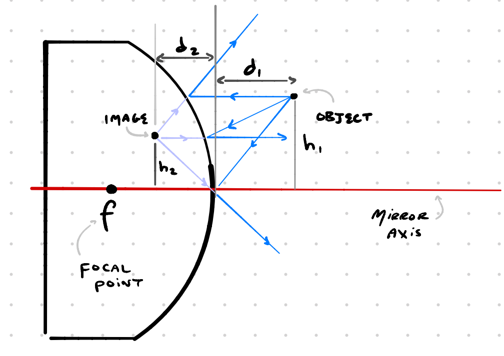
From the mirror equation:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

$$\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} < 0$$

negative for convex mirror

positive by convention



CONVEX MIRROR

$\longrightarrow$ $d_2 < 0$ for convex mirrors

Mirror Conventions

- So d_2 negative: image and object on opposite sides of mirror.

[Contrast this w/ convention for lenses]

Since $h_2 = -\frac{h_1}{d_1} d_2 > 0$ we also have:

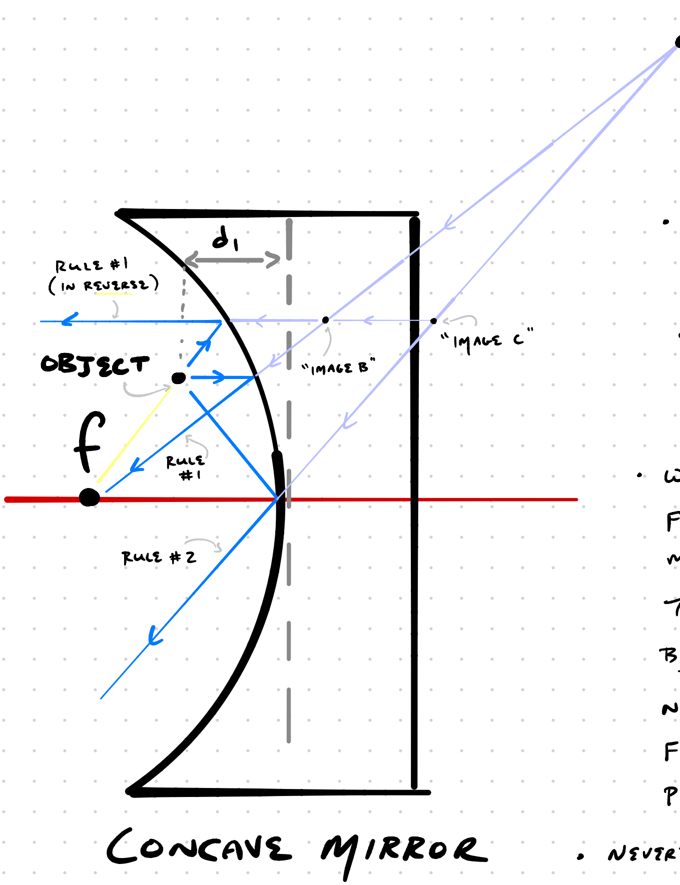
positive by convention

negative for convex mirrors

- h_2 positive: image & object on same side of mirror axis.

[Same as for lenses]

b)



OBSERVATIONS

- FROM THREE RAYS, DETERMINED USING RULES #1, #2, & #1 IN REVERSE, WE HAVE THREE PAIRS OF RAYS THAT ALL INTERSECT @ DIFFERENT POINTS (A, B, C)!
- WE FIND THAT IMAGES FORMED BY RAYS THAT MAKE LARGE ANGLES W/ THE MIRROR AXIS (E.G. RULE #2 & RULE #1 IN REVERSE) ARE BLURRY*, I.E. THE RAYS DO NOT ALL APPEAR TO COME FROM A SINGLE IMAGE POINT.
- NEVERTHELESS, THE IMAGE, HOWEVER BLURRY, DOES APPEAR ✓UPRIGHT [OBJECT & IMAGE ON SAME SIDE OF MIRROR AXIS] AND IS VIRTUAL✓ [RAYS ONLY APPEAR TO ORIGINATE FROM IMAGE LOCATION].
- THIS IS CONSISTENT W/ MIRROR EQUATION: $\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} < 0$ $(d_1 < f)$

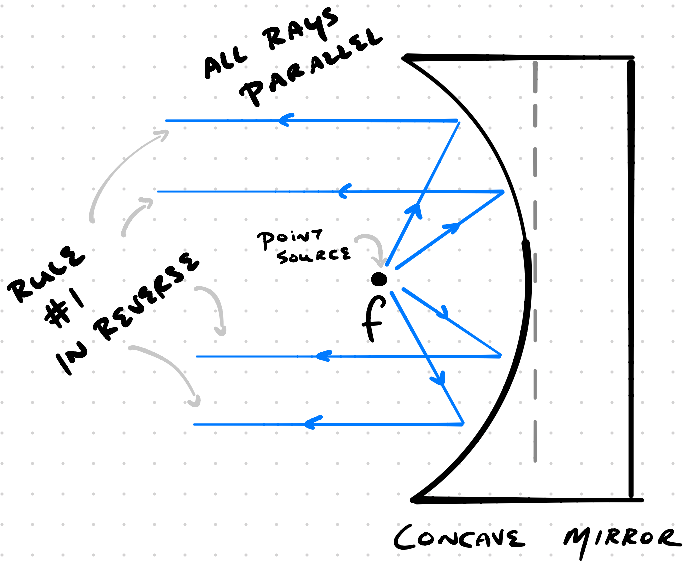
$\rightarrow d_2 < 0$, WHICH MEANS IMAGE IS UPRIGHT & VIRTUAL.

- CENTER OF BLURRY IMAGE (d_2) IS GIVEN BY MIRROR EQUATION:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

* WE SAY THE IMAGE IS HIGHLY "ABERRATED".

c) PUT @ FOCAL POINT:



Q3

a) $f = 2''$, $d_2 = 4''$

$$\frac{1}{d_1} = \frac{1}{f} - \frac{1}{d_2}$$

$$= \frac{1}{2''} - \frac{1}{4''} = \frac{1}{4''}$$

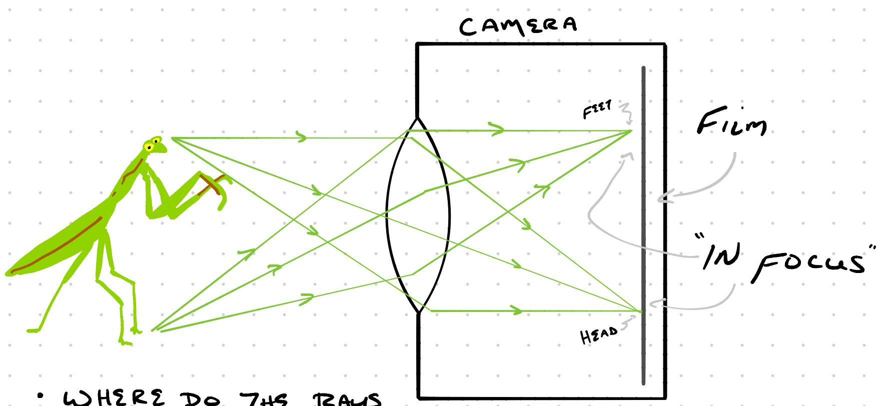
$$\longrightarrow d_1 = 4''$$

$h_1 = 2''$ (BUG HEIGHT)

$$h_2 = -\frac{d_2}{d_1} \times h_1 = -\frac{4''}{4''} \times 2'' = -2''$$

FILM NEEDS TO BE AT LEAST THIS LARGE

(I.E. 1× MAGNIFICATION, INVERTED)

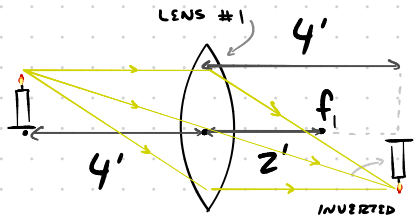


- WHERE DO THE RAYS COME FROM?
- WHY ARE THEY GREEN?

b) IMAGE OF LENS #1:

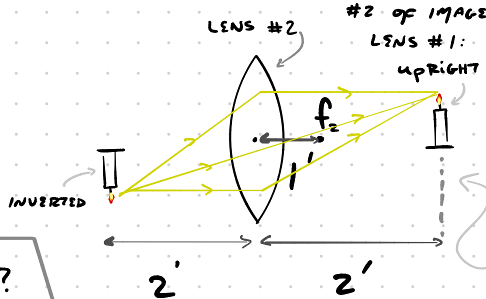
$$\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} = \frac{1}{2'} - \frac{1}{4'} = \frac{1}{4'}$$

$$d_2 = 4'$$



- IMAGE OF LENS #2:

$d_1 = 6' - 4' = 2'$, $\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} = \frac{1}{1'} - \frac{1}{2'} = \frac{1}{2'} \longrightarrow d_2 = 2'$



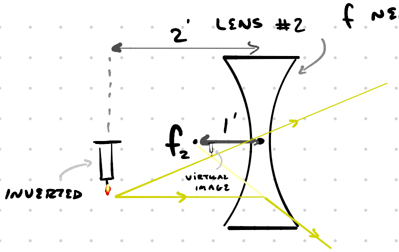
PLACE SCREEN HERE WHERE RAYS FROM CANDLE ARE CONVERGING, I.E. X = 2'

+ IF LENS #2 DIVERGING?

- IMAGE FROM LENS #1 IS THE SAME, SO

$$\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} = \frac{1}{-1'} - \frac{1}{2'} = -\frac{3}{2'} \longrightarrow d_2 = -\frac{2}{3}'$$

f NEGATIVE!



RAYS NEVER CONVERGE ANYWHERE!

CAN'T USE SCREEN TO VIEW VIRTUAL IMAGE!

c) d_1 : DISTANCE FROM MOON TO LENS

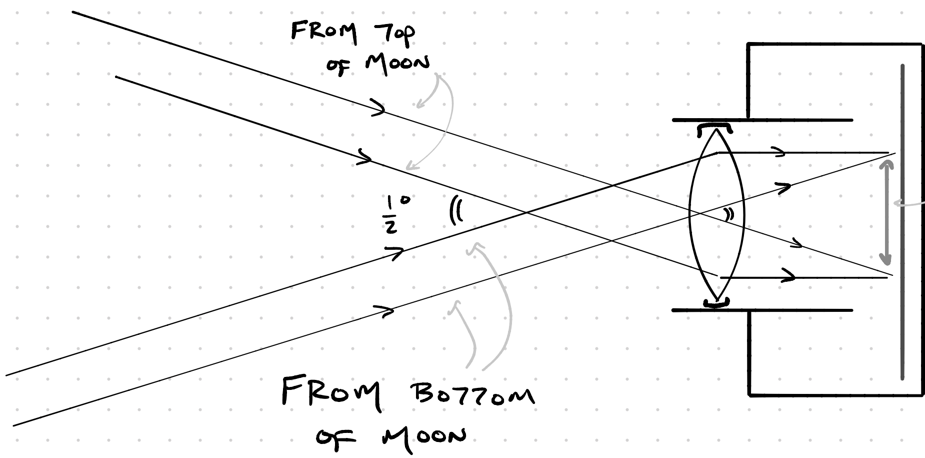
f : FOCAL LENGTH $= +5'' \ll d_1$

MOON IN FOCUS WHEN $x = d_2$

WHERE $$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

$$\longrightarrow \frac{1}{d_2} = \frac{1}{f} - \cancel{\frac{1}{d_1}}^{\,0} \quad (d_1 \gg f)$$

$$\longrightarrow d_2 = f, \text{ SO SET } x = 5'' ✓$$



SPOT SIZE

$= 2f \tan \theta/2$

(GEOMETRY)

$= 2 \times 5'' \times \tan \frac{1}{4}°$

$\approx 2 \times 5'' \times \frac{1/4°}{180°} \pi$

(SMALL ANGLE APPROX)

$= .04'' \approx 1\,mm$ ✓