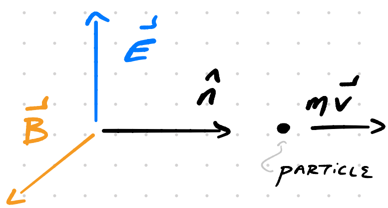
LECTURE 23 QUESTIONS

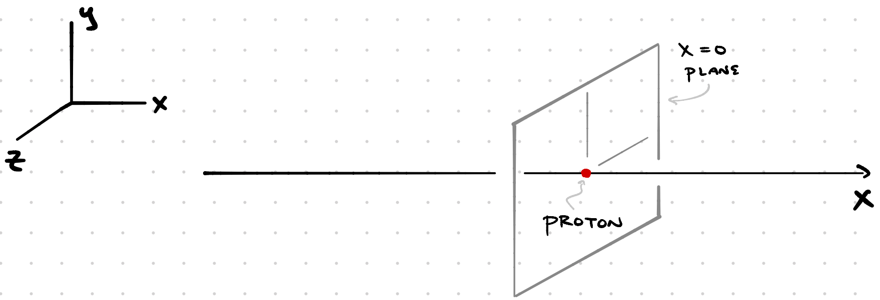
E.M. WAVES + MOMENTUM

- IN QUESTIONS #1 + #2 WE WILL LOOK @ HOW AN E.M. WAVE CAN TRANSFER MOMENTUM TO PARTICLES ALONG THE DIRECTION $\hat{n}$ OF THE WAVE'S PROPAGATION, EVEN THOUGH THE $\vec{E}$ + $\vec{B}$ FIELDS ARE BOTH $\perp$ TO $\hat{n}$:

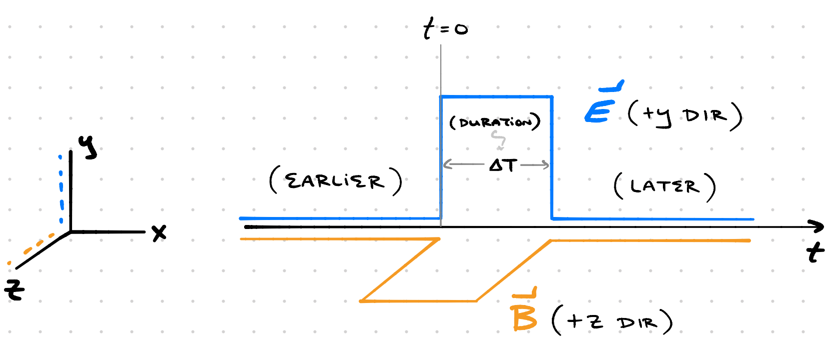


Q1

A proton is sitting @ rest at some point (x = 0, y = 0, z = 0) in space:



- Consider an E.M. wave that is shaped like a rectangular pulse traveling in the +x direction, so that anywhere in the x = 0 plane, the $\vec{E}$ & $\vec{B}$ fields have the following time dependence:



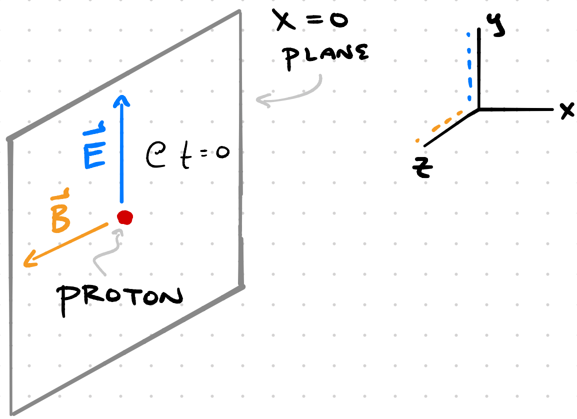
IN CASE YOU FORGOT :)

$\vec{F}_{\vec{E}} = q \times \vec{E}$

$F_{\vec{B}} = q \times v \times B \times \sin\theta$, $\vec{F}_{\vec{B}} \perp \vec{v}, \vec{B}$ (RHR #1)

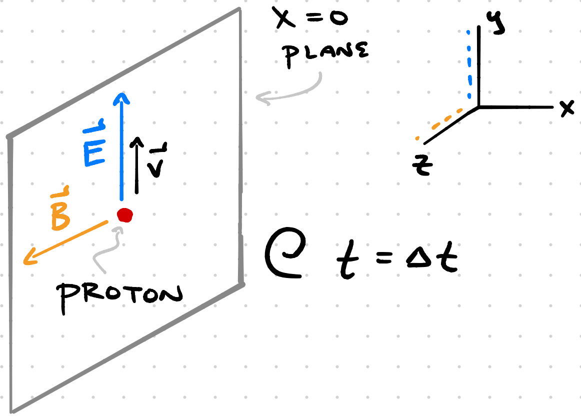
a) @ $t = 0$, THE FORCE ON THE PROTON COMES ENTIRELY FROM THE $\vec{E}$ FIELD.

- WHY IS THERE NO FORCE APPLIED BY THE $\vec{B}$ FIELD @ $t = 0$?
- WHAT IS THE DIRECTION OF THE FORCE ON THE PROTON @ $t = 0$?



- WHAT DIRECTION WILL THE PROTON'S VELOCITY BE SOME SHORT TIME Δt LATER AS A RESULT OF THIS FORCE?

b) Some short time $\Delta t < \Delta T$ (pulse duration) later, the proton is moving in the $+y$ direction parallel to $\vec{E}$:



- The force exerted by the $\vec{B}$ field is now non-zero [why?].

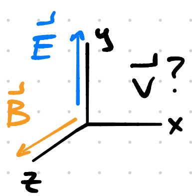
- In which direction does this force push the proton (R.H.R. #1)?

Q2

a) From the relation $B = \frac{E}{c}$ & the force laws for $\vec{E}$ & $\vec{B}$, determine the ratio $\frac{F_{\vec{B}}}{F_{\vec{E}}}$ of the force on the proton from the magnetic & electric fields of an E.M. wave.

- Answer is in terms of c & v. (speed of light: c; particle's speed: v)

b) What direction must a proton* be travelling so that these forces point in opposite directions?

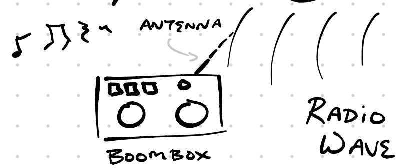


c) How fast must the proton be going if these forces perfectly cancel? [Assume proton moves in direction from part (b)]

* Does your answer change if we instead consider an electron?

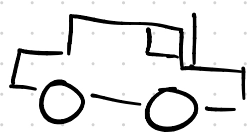
Q3 ELECTROMAGNETIC Spectrum

a) AN ANTENNA IS A STRUCTURE WHICH ABSORBS ELECTROMAGNETIC RADIATION [WAVES].

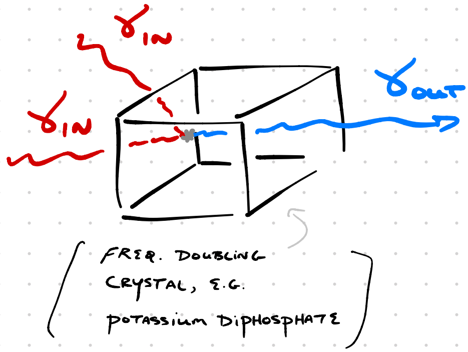


- ABSORPTION IS MOST EFFICIENT WHEN THE LENGTH OF THE ANTENNA IS THE SAME ORDER OF MAGNITUDE AS THE WAVELENGTH OF RADIATION BEING ABSORBED.

- HOW LONG SHOULD YOU DESIGN AN ANTENNA FOR PICKING UP F.M. RADIO, WHICH TRANSMITS @ ~ 100 MHz?
  - COMPARE w/ LENGTH OF TYPICAL CAR HOOD ANTENNA:

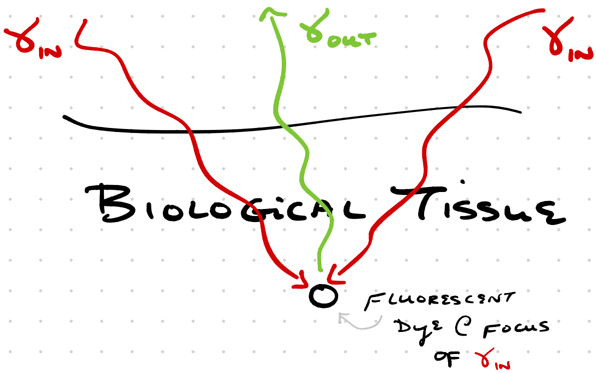


b) A "FREQUENCY DOUBLING CRYSTAL" CAN TAKE TWO PHOTONS OF ONE COLOR AND OUTPUT A PHOTON WHOSE ENERGY EQUALS THE SUM OF THE ORIGINAL TWO:



- IF PHOTON ENERGY IS PROPORTIONAL TO FREQUENCY, THEN WHAT IS THE WAVELENGTH OF THE OUTPUT OF OF A FREQ. DOUBLING CRYSTAL THAT IS GIVEN 1000 nm [INFRARED] LIGHT AS INPUT? IS IT VISIBLE?

c) Two-Photon Microscopy

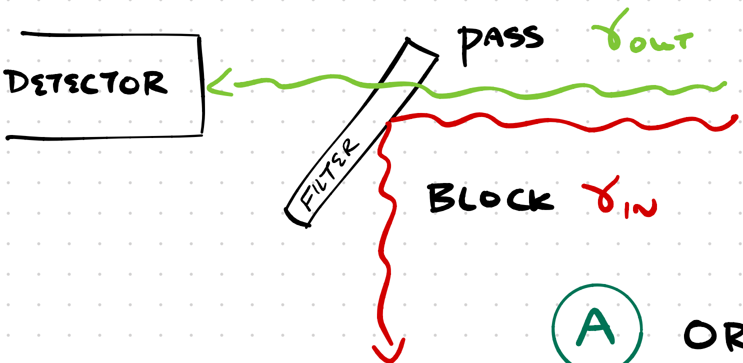


- Two red photons simultaneously absorbed by fluorescent dye.

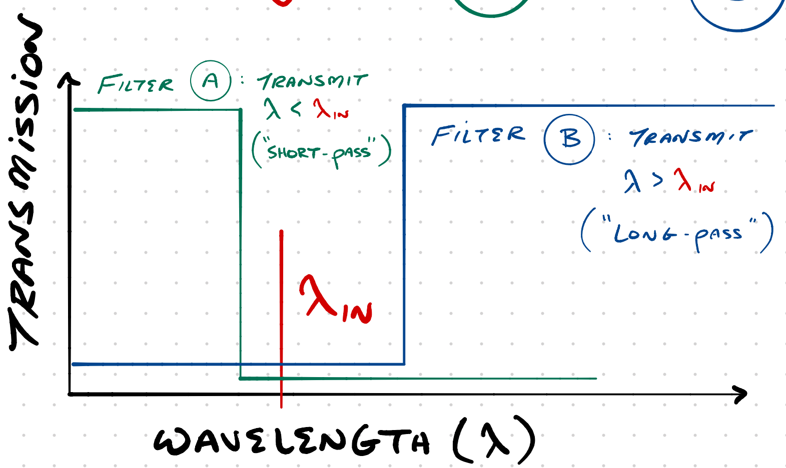
- Collect photon (γ_{OUT}) emitted by dye. Scan focus of input light (γ_{IN}), monitoring output (γ_{OUT}) to construct image of dye concentration in biological tissue.

- IF THE ENERGY OF THE OUTPUT PHOTON γ_{OUT} EMITTED BY THE EXCITED DYE MOLECULES IS <u>GREATER</u> THAN THE INPUT PHOTONS γ_{IN}, THEN WHAT FILTER SHOULD WE PUT ON THE DETECTOR SO THAT WE BLOCK THE γ_{IN}^{*} BUT PASS THE γ_{OUT}?

[RECALL FROM PART (b) THAT PHOTON ENERGY $\propto$ FREQUENCY]



(A) OR (B) ?



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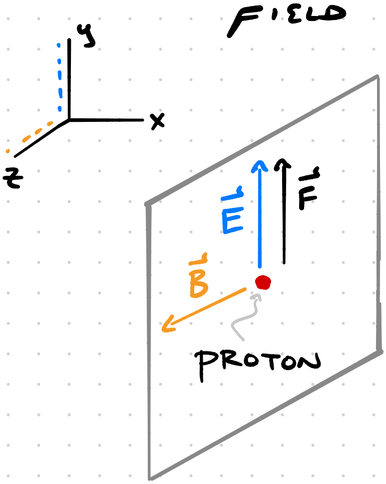
* THESE γ_{IN} ARE UNDESIRABLE "BACKGROUND" COMING FROM RANDOM SCATTERING IN THE BIOLOGICAL TISSUE. THE γ_{IN} ARE TYPICALLY CHOSEN TO BE INFRARED SINCE THESE WAVELENGTHS CAN PENETRATE FAR ($\simeq$ 1 cm) INTO TISSUES W/OUT SCATTERING.

ANSWERS

Q1 a)

- $F_{\vec{B}} \propto v$, which is still zero @ $t=0$ before the wave arrives.

- Force on positive charge by $\vec{E}$ field is $\parallel$ to $\vec{E}$:



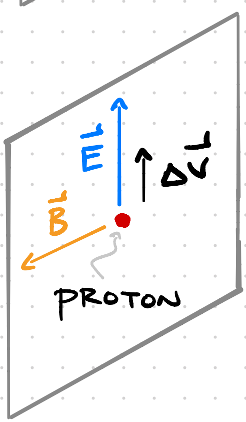
@ $t=0$

- @ $t=\Delta t$,

acceleration due to $\vec{F}$ produces velocity $\parallel \vec{F}$:

$$\vec{F} = m\vec{a}$$

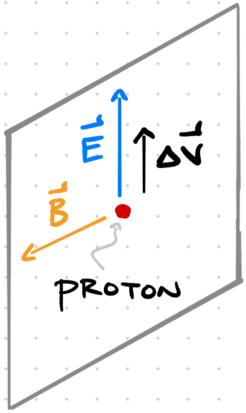
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

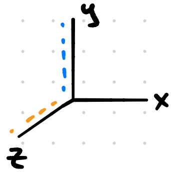


@ $t=\Delta t$

$$\longrightarrow \Delta\vec{v} = \frac{\vec{F}}{m}\Delta t$$

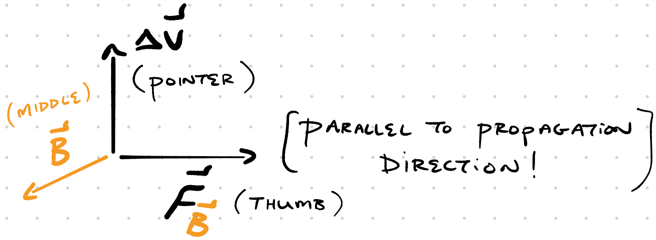
b)





$@\ t = \Delta t$

R.H.R. #1 :

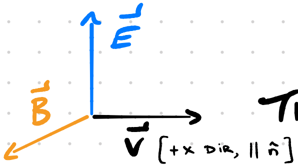


- $\vec{E}$ AND $\vec{B}$ FORCES CONSPIRE TO PUSH CHARGE ALONG THE E.M. WAVE!

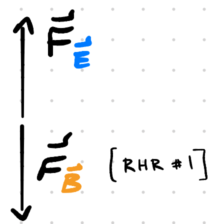
[I.E. PARALLEL TO $\hat{n}$]

Q2 a) $$\frac{F_{\vec{B}}}{F_{\vec{E}}} = \frac{(+e)\,v\,B}{(+e)\,E} = \frac{v\,E/c}{E} = \frac{v}{c}$$

b)



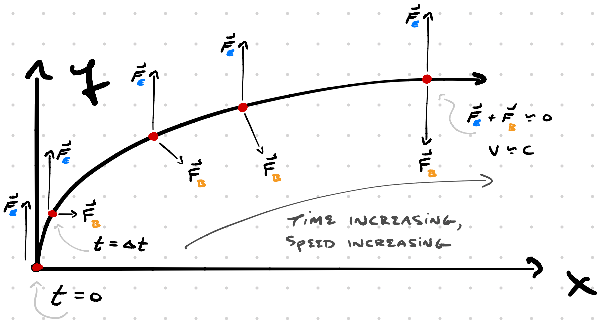
THEN :



c) $F_B = F_E$ WHEN $v = c$ [PART (a)]

Q1 & Q2 wrap up:

- So if E.M. pulse is strong enough, proton ends up moving parallel to wave w/ velocity approaching c:



Q3

a) $f\lambda = c \rightarrow \lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \, m/s}{100 \, MHz}$

$= \frac{3 \cdot 10^8 \, m/s}{10^2 \times 10^6 \, Hz}$

$= 3m$

COMPARABLE TO CAR ANTENNA.

b) ENERGY $\propto f$

$= hf$

"PLANCK'S CONSTANT"

SO IF $E_2 = E_1 + E_1 = 2E_1$

AND $\lambda_1 = 1000 \, nm$, THEN:

$$\lambda_2 = \frac{c}{f_2} = \frac{hc}{E_2} = \frac{hc}{2E_1} = \frac{1}{2} \times \frac{hc}{E_1} = \frac{1}{2} \times \frac{c}{f_1} = \frac{1}{2} \times \lambda_1 = 500 \, nm$$

- VISIBLE WAVELENGTHS ARE $\simeq$ 400nm - 750nm
- 500nm IS GREENISH BLUE

c) IF γ_{OUT} PHOTON IS HIGHER ENERGY THAN γ_{IN}, THEN $\lambda_{OUT} < \lambda_{IN}$. (PART (b))

- THEREFORE, WE WANT OUR FILTER TO PASS $\lambda < \lambda_{IN}$ (IE FILTER (A)).