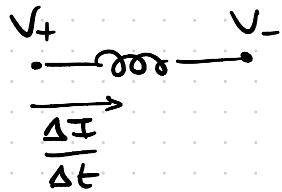
LECTURE 22 QUESTIONS

Q1

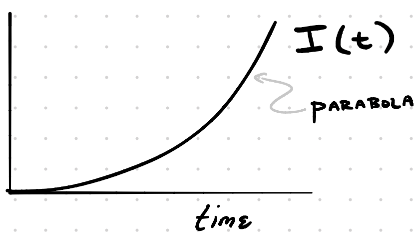
a) IF THE CURRENT THROUGH AN INDUCTOR IS INCREASING @ A RATE OF $1 \frac{mA}{\mu s}$, AND THE INDUCTOR HAS AN INDUCTANCE $L = 500\,\mu H$, WHAT IS THE VOLTAGE ACROSS THE INDUCTOR?



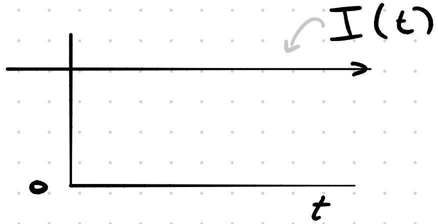
b) BELOW IS THE PLOT OF THE CURRENT $I(t)$ THRU AN INDUCTOR VS. TIME.

PLOT THE VOLTAGE ACROSS THE INDUCTOR ON THE SAME GRAPH:

$I(t) = \frac{1}{2} a t^2$



c) PLOT V(t):



d) EXPLAIN HOW PART (c) DEMONSTRATES THE PRINCIPLE, "IN STEADY STATE AN INDUCTOR ACTS LIKE A SHORT-CIRCUIT."

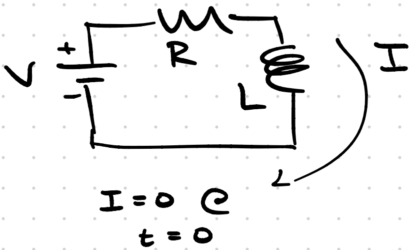
e) IS THIS PRINCIPLE CONSISTENT W/ WHAT YOU KNOW INDUCTORS TO BE MADE OF?

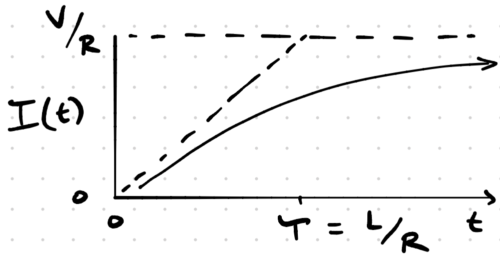
BONUS: f) TWO INDUCTORS OF EQUAL INDUCTANCE ARE DRIVEN W/ A.C. CURRENTS OF EQUAL AMPLITUDES BUT DIFFERENT FREQUENCIES $f_1 < f_2$.

- WHICH HAS THE LARGER INDUCED VOLTAGE?

Q2 SERIES RL SERIES CIRCUIT

REFER TO THE RESULT WORKED OUT FOR THE CURRENT IN THE RL SERIES CIRCUIT:





a) USE OHM'S LAW TO FIND:

- V_R @ $t = 0$
- V_R AS $t \rightarrow \infty$

b) USE K.V.L. TO FIND

- V_L @ $t = 0$
- ** V_L AS $t \rightarrow \infty$

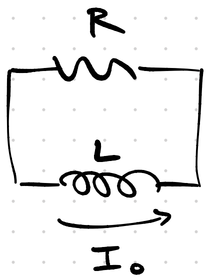
* IS THIS CONSISTENT W/ THE PRINCIPLE, STATED IN Q1, THAT,

"IN STEADY-STATE AN INDUCTOR ACTS LIKE A SHORT CIRCUIT."

* COMPARE THIS W/ THE PREDICTION FROM THE DEFINING RELATIONSHIP FOR AN INDUCTOR: $V = L\frac{\Delta I}{\Delta t}$

Q3) Discharging the RL circuit

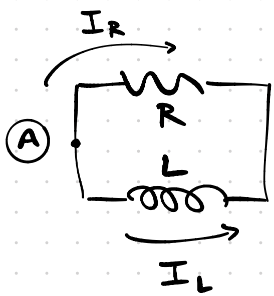
- @ time $t=0$, suppose an inductor in an inductor + resistor combo carries a current I_0, as shown below*:



@ $t=0$

- Let's work out how the inductor current changes as time goes on.

a)



Apply the K.C.L. @ the node A to show that:

$$I_R = -I_L$$

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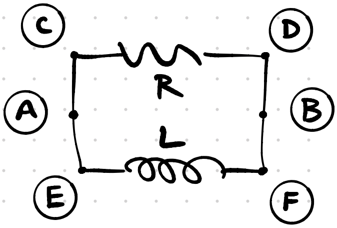
* This could describe, for instance, an aluminum ring w/ inductance L + resistance R.

b) 

• USE THE K.V.L. FOR THE PATHS $A \to C \to D \to B$ + $A \to E \to F \to B$ TO SHOW THAT:

$$V_R = V_L$$

c) USE OHM'S LAW AND THE DEFINITION OF INDUCTANCE TO SHOW THAT FROM (b):

$$I_R R = L \frac{\Delta I_L}{\Delta t}$$

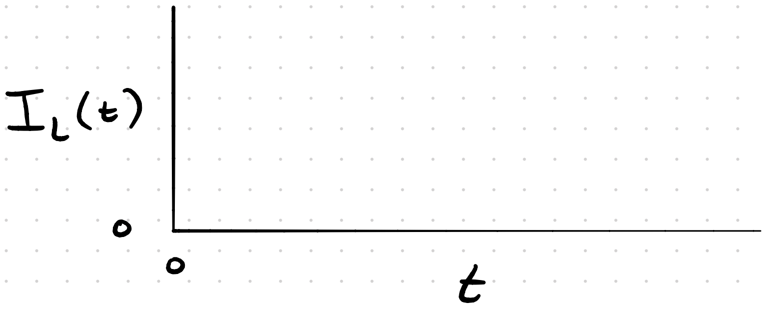
d) COMBINE RESULT FROM PART a) w/ RESULT FROM PART c) TO SHOW: A

$$\boxed{\frac{\Delta I_L}{\Delta t} = \frac{-I_L}{\tau}} \quad \text{WHERE } \tau = \frac{L}{R}$$

e) A IS WHAT WE WILL NEED TO FIGURE OUT $I_L(t)$ FOR $t > 0$. FOR STARTERS, THOUGH, SIMPLY MAKE A POINT ON THE I_L VS. t GRAPH FOR THE INDUCTOR CURRENT I_L @ $t = 0$:

(SEE NEXT PAGE):

(e) CONT. : MARK $I_L(t)$ @ $t=0$:



f) USING A + (e), DETERMINE THE RATE OF CHANGE $\frac{\Delta I_L}{\Delta t}$ @ $t=0$.

- NOTE THE SIGN OF THE SLOPE.
- DRAW A LINE STARTING FROM THE POINT YOU MARKED IN (e), EXTENDING TO THE HORIZONTAL (t) AXIS.
- WHERE DOES IT INTERSECT THE HORIZONTAL AXIS?

g) NOW, USE A, ALONG W/ THE "STEADY STATE CONDITION" (NO CHANGING CURRENTS OR VOLTAGES), TO PREDICT THE INDUCTOR CURRENT I_L AS $t \to \infty$.

- DRAW A HORIZONTAL LINE THAT INTERSECTS THE VERTICAL (I_L) AXIS AT THIS VALUE.

h) FINALLY, DRAW A SMOOTH CURVE THAT:

- STARTS @ THE POINT MARKED IN STEP (e), THEN
- MOVES INITIALLY ALONG THE LINE DRAWN IN PART (f), THEN:
- "PEELS OFF" THIS LINE, AND GRADUALLY APPROACHES THE LINE FROM PART (g) AS TIME GOES ON ($t \to \infty$).

(i) DO YOU RECALL THE NAME OF THIS CURVE? MATHEMATICALLY IT HAS THE FORM

$$\boxed{I_L(t) = I_0 e^{-t/\tau}}$$

- CHECK THAT THIS EQUATION AGREES W/ YOUR ANSWERS FOR PARTS (e) & (g), I.E. @ $t=0$ & $t \to \infty$.
- WHEN DOES THIS FORMULA PREDICT THAT $I_L(t)$ GOES TO $\frac{1}{e}$ OF ITS INITIAL VALUE? DOES THIS (ROUGHLY) AGREE W/ YOUR GRAPH?

$$(e \doteq 2.72)$$

Bonus! · WHEN DOES $I_L(t)$ GO TO $1/2$ OF ITS INITIAL VALUE?

ANSWERS

Q1

a) $V = L \dfrac{\Delta I}{\Delta t}$

$= 500\,\cancel{\mu}H \cdot \dfrac{1\,mA}{\cancel{\mu}s}$

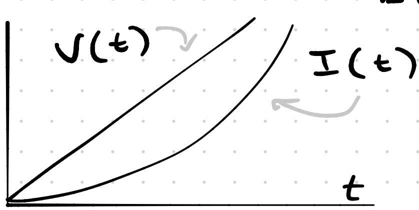
$= 500\,mV$

$= .5\,V$

b) ANALOGY W PROJECTILE MOTION:

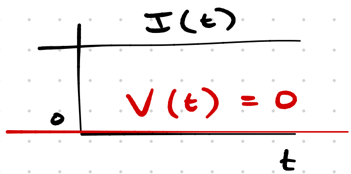
- IF $x(t) = \frac{1}{2}at^2$
- THEN $v(t) = at$
- SO IF $I(t) = \frac{1}{2}at^2$
- THEN $\dfrac{\Delta I(t)}{\Delta t} = at$

$\longrightarrow V(t) = L \dfrac{\Delta I(t)}{\Delta t} = Lat$



c) CURRENT CONSTANT $\rightarrow \dfrac{\Delta I}{\Delta t} = 0$

$\rightarrow V = 0$

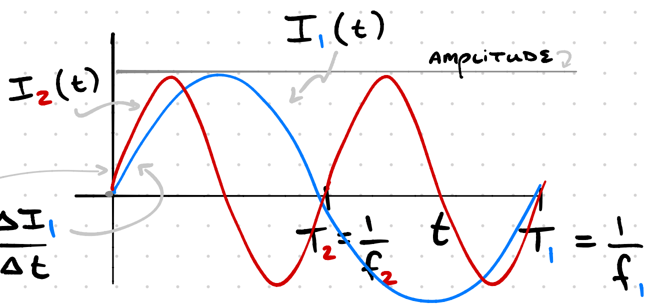


d) IN (C) WE FIND THAT IN STEADY-STATE, WHERE CURRENTS ARE NOT CHANGING, THE VOLTAGE ACROSS AN INDUCTOR IS ZERO, NO MATTER WHAT (STEADY!) CURRENT CONDUCTS THRU IT.

THIS "$V = 0$ FOR ANY I" BEHAVIOR IS JUST HOW SHORT-CIRCUITS BEHAVE*.

e) WELL, INDUCTORS ARE MADE OF (CONDUCTING) WIRE, SO IT IS NOT SURPRISING THAT THEY ACT LIKE SHORT-CIRCUITS (WHEN CURRENT IS STEADY).

f)



$$\frac{\Delta I_2}{\Delta t} > \frac{\Delta I_1}{\Delta t}$$

$$\longrightarrow V_2 = L\frac{\Delta I_2}{\Delta t} > L\frac{\Delta I_1}{\Delta t} = V_1$$

Q2

FROM GRAPH:

- $I = 0$ @ $t = 0$ [OUR INITIAL ASSUMPTION]
- $I \to V/R$ AS $t \to \infty$ ["STEADY-STATE"]

a) FROM OHM'S LAW:

$$(V_R = IR)$$

- $V_R = (0) \times R = 0$ @ $t = 0$
- $V_R \longrightarrow V/R \times R = V$ AS $t \to \infty$

b) K.V.L.: $V = V_R + V_L \to V_L = V - V_R$

SO:

- $V_L = V - (0) = V$ @ $t = 0$
- ** $V_L \longrightarrow V - (V) = 0$ AS $t \to \infty$.

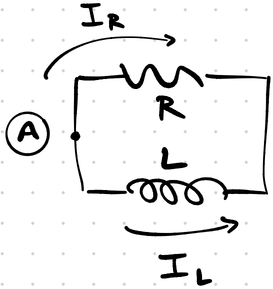
* AGAIN WE SEE THAT VOLTAGE ACROSS INDUCTOR IS ZERO IN STEADY STATE, EVEN THOUGH CURRENT IS NON-ZERO ($I \to V/R$), I.E. INDUCTOR ACTS LIKE SHORT-CIRCUIT.

* WE ALSO SEE SLOPE OF $I(t)$ FLATTENING OUT AS $t \to \infty$, I.E. $\frac{\Delta I}{\Delta t} \longrightarrow 0$. SO AGAIN WE FIND

$V_L = L \frac{\Delta I}{\Delta t} \longrightarrow 0$ IN STEADY STATE.

Q3

a)

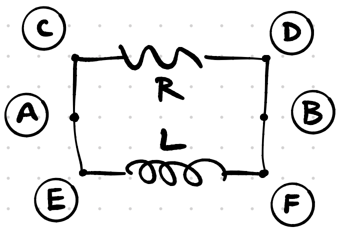


CURRENT INTO A = CURRENT OUT OF A

$$0 = I_L + I_R$$

$$\longrightarrow \quad I_R = -I_L$$ ✓

b)



KVL: $$V_{A\to C\to D\to B} = V_{A\to E\to F\to B}$$

(wires)

$$\cancelto{0}{V_{A\to C}} + V_{C\to D} + \cancelto{0}{V_{D\to B}} = \cancelto{0}{V_{A\to E}} + V_{E\to F} + \cancelto{0}{V_{F\to B}}$$

$$V_R = V_L$$

$$\longrightarrow \quad V_R = V_L$$ ✓

c) OHM'S LAW : $V_R = I_R R$

INDUCTORS : $V_L = L \frac{\Delta I_L}{\Delta t}$

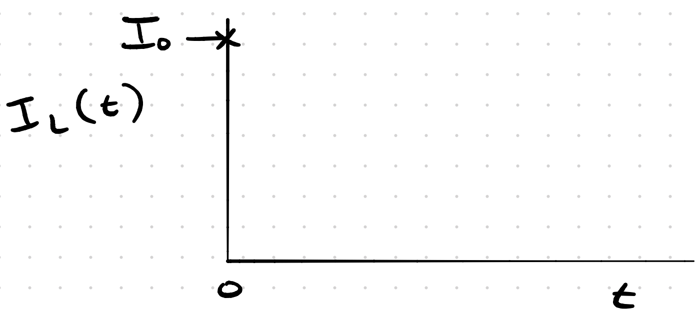
$$I_R R = V_R \underset{(b)}{=} V_L = L \frac{\Delta I_L}{\Delta t}$$

d) PLUGGING IN [FROM (a)] $I_R = -I_L$ INTO (c) WE GET :

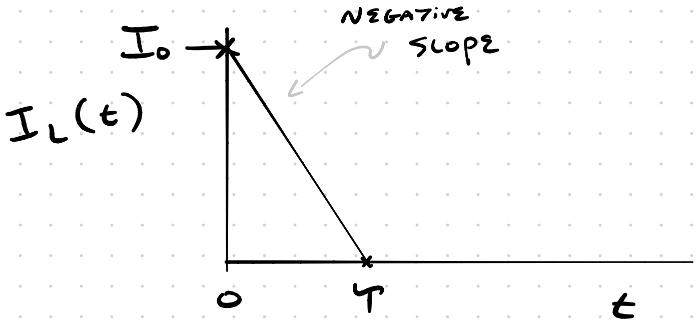
$$-I_L R = L \frac{\Delta I_L}{\Delta t}$$

$$\longrightarrow \frac{\Delta I_L}{\Delta t} = -I_L \times \frac{R}{L} = -\frac{I_L}{L/R} = -\frac{I_L}{T}$$

e) By <u>ASSUMPTION</u> : $I = I_0$ @ $t = 0$



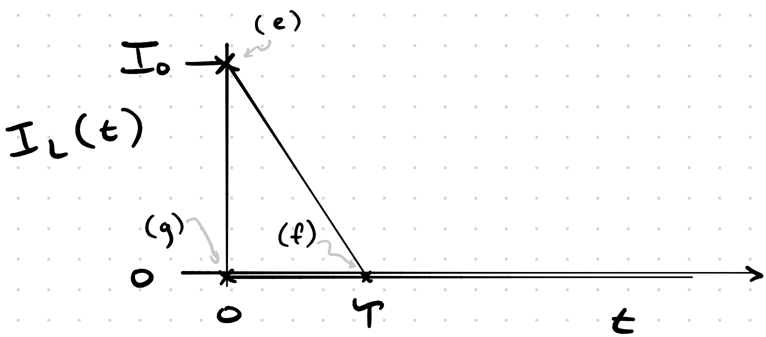
f) A $\dfrac{\Delta I_L}{\Delta t} = -\dfrac{\cancel{I}^{\,I_0 \,@\, t=0}}{T} = -\dfrac{I_0}{T}$



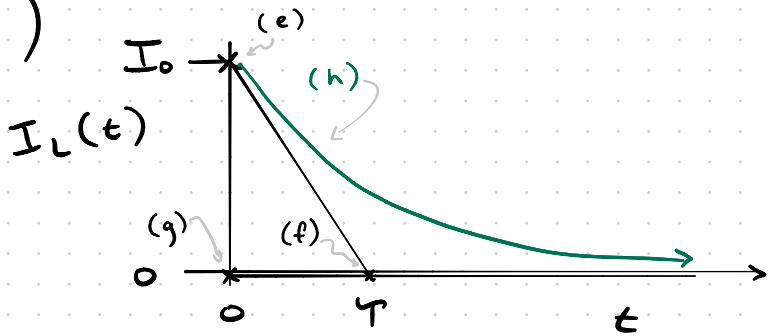
g) STEADY-STATE : $\dfrac{\Delta I_L}{\Delta t} \longrightarrow 0$

$\longrightarrow$ A $\cancelto{0}{\dfrac{\Delta I_L}{\Delta t}} = -\dfrac{I}{T}$

$\longrightarrow \quad I \longrightarrow 0$



h)



i) "EXPONENTIAL DECAY"

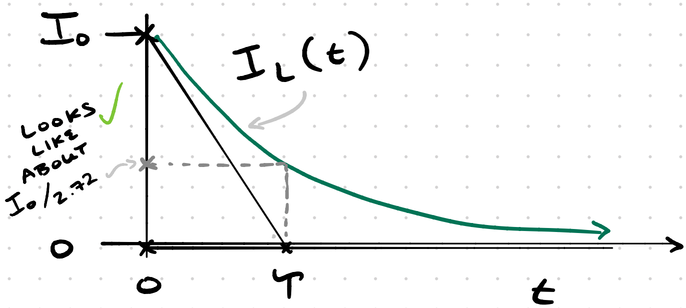
@ $t = 0$ $\qquad I(t=0) = I_0 e^{-0/\tau} = I_0 \cancel{e}^{\,0 \to 1} = I_0$ ✓

AS $t \to \infty$ $\qquad I(t\to\infty) = I e^{-\infty/\tau} = \dfrac{I}{e^{\infty/\tau}} \to 0$ ✓

(DENOM. GETS BIG!)

- I_L GOES TO $\dfrac{I_0}{e}$ @ $t = \tau$

$$I(t=\tau) = I_0 e^{-\tau/\tau} = I_0 e^{-1} = \frac{I_0}{e} \simeq \frac{I_0}{2.72}$$



- @ WHAT $t_{1/2}$ DOES $I(t) = I_0/2$?

$$I(t_{1/2}) = I_0 e^{-t_{1/2}/\tau} = I_0/2$$

$$\frac{I_0}{e^{t_{1/2}/\tau}} = \frac{I_0}{2}$$

$$\longrightarrow \quad e^{t_{1/2}/\tau} = 2$$

$$\ln\left(e^{t_{1/2}/\tau}\right) = \ln 2$$

$$t_{1/2}/\tau = \ln 2$$

$$t_{1/2} = \ln 2 \times \tau \simeq .69\tau$$