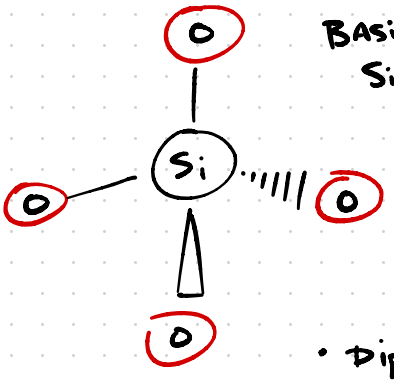
Lecture 28 Notes

Birefringence.

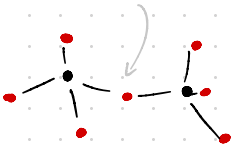
BIREFRINGENCE

- SOME MATERIALS HAVE A POLARIZATION-DEPENDENT INDEX OF REFRACTION
- E.G. QUARTZ (CRYSTALLINE SiO_2):
  - STRUCTURE:

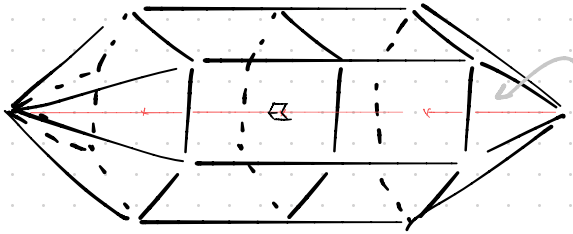


BASIC UNIT: SiO_4 TETRAHEDRON

- DIFFERENT UNITS SHARE OXYGEN-CORNERS:

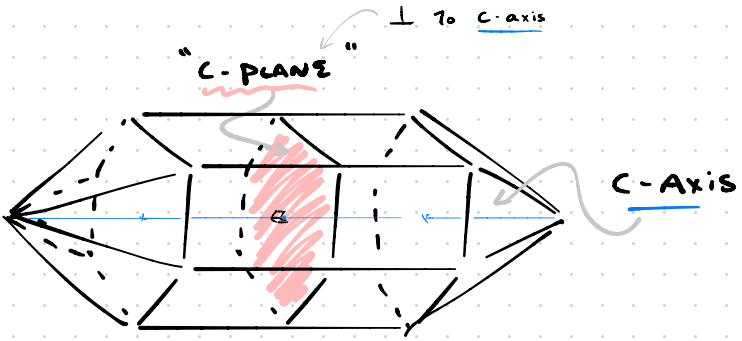


- QUARTZ CRYSTALS GROW AS HEXAGONAL RODS:

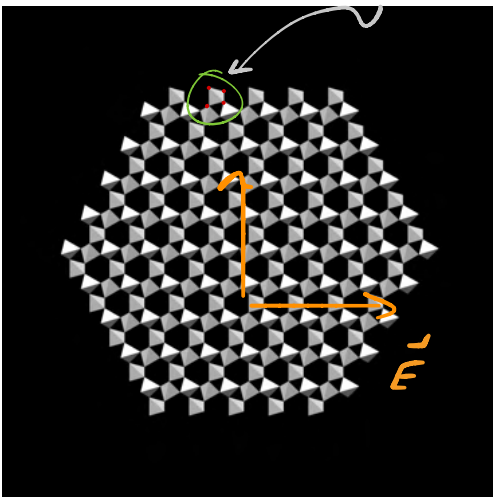


· (QUARTZ CONT.) MICROSCOPIC STRUCTURE

IS ASYMMETRIC : HAS DISTINGUISHABLE PLANES :



SiO_4 UNIT



C-PLANE :

INDEX of REFRACTION

$n_o = 1.544$ FOR

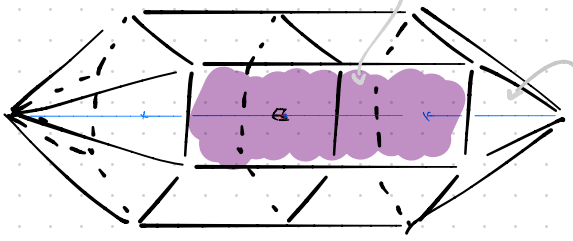
(o ~ "ORDINARY") LIGHT

POLARIZED IN C-PLANE

· (QUARTZ CONT.) MICROSCOPIC STRUCTURE

IS ASYMMETRIC : HAS DISTINGUISHABLE PLANES :

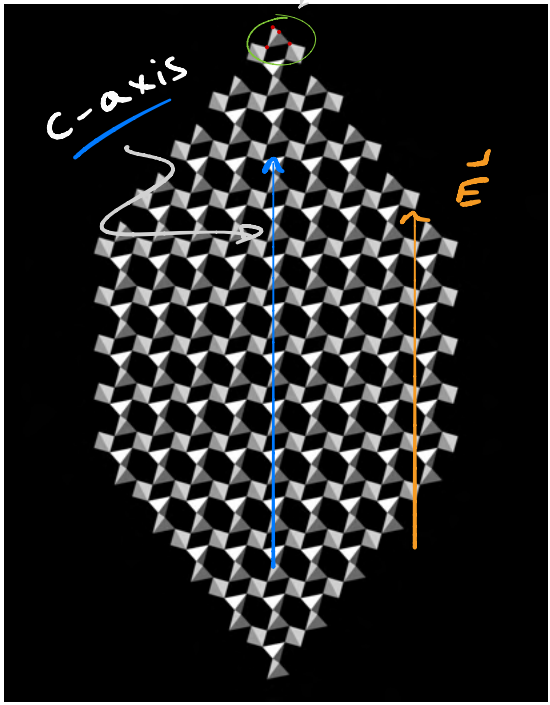
a - PLANE (∥ TO C-axis)



NOTE HOW SiO_4 UNITS "LINK UP" DIFFERENTLY ALONG a-PLANE VS. C-PLANE .

(PREV. PAGE)

SiO_4 UNIT



a - PLANE

INDEX OF REFRACTION

$n_e = 1.554 \neq n_o$

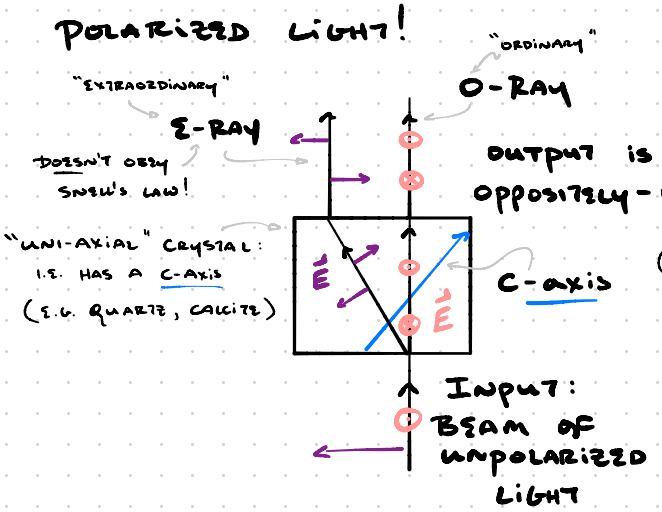
(e = "EXTRAORDINARY")

FOR LIGHT POLARIZED

∥ TO C-axis

DOUBLE REFRACTION

- UNPOLARIZED LIGHT SPLITS INTO TWO BEAMS OF POLARIZED LIGHT!



OUTPUT IS TWO OPPOSITELY-POLARIZED BEAMS!

(DOUBLE REFRACTION MOST PRONOUNCED WHEN C-AXIS @ 45° TO SURFACE NORMAL. EFFECT IS ABSENT @ 0° OR 90°.)

CALCITE ($CaCO_3$)

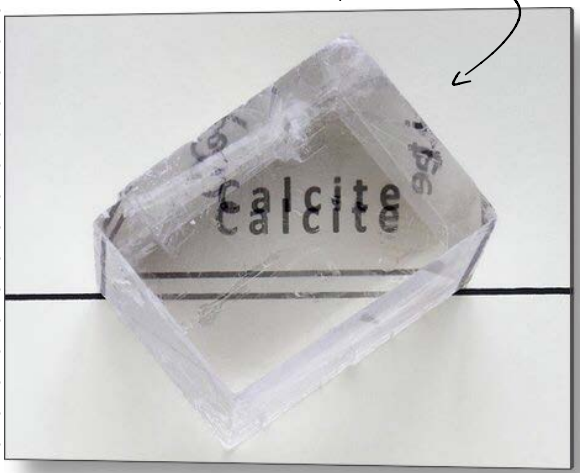
VERY BIREFRINGENT:

$n_o = 1.658$

$n_e = 1.486$

$n_o - n_e = 0.17$

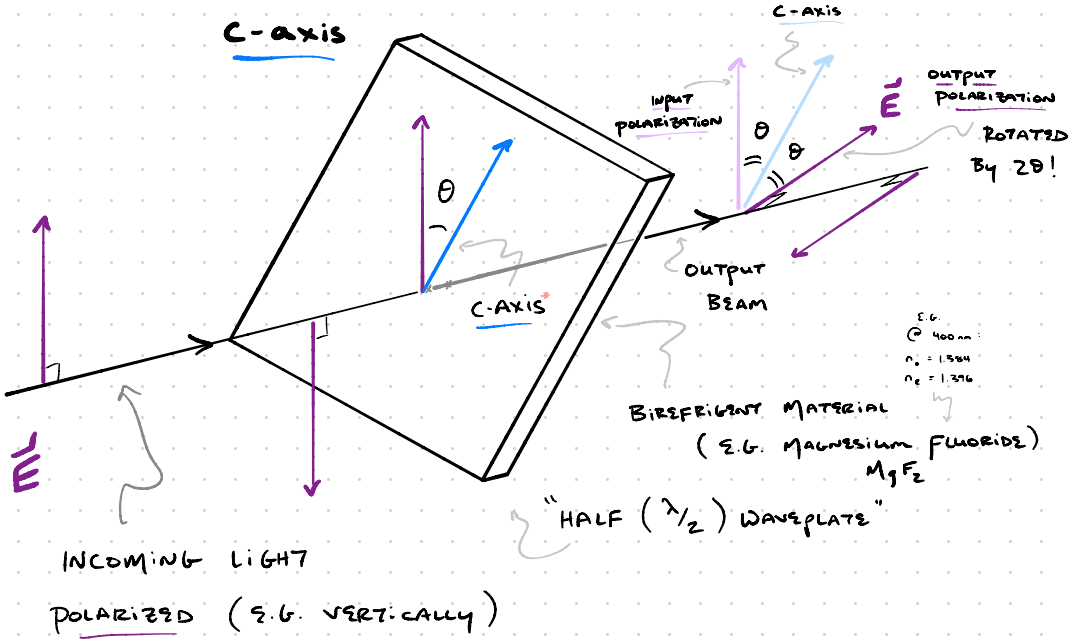
(COMPARE TO QUARTZ: $n_e - n_o = 0.01$)



NOT TYPICALLY TRANSPARENT ("ICELAND SPAR")

WAVEPLATES

- BIREFRINGENT MATERIALS CAN BE USED TO MANIPULATE THE POLARIZATION OF LIGHT:
  - E.G. HALF-WAVE PLATE (HWP):

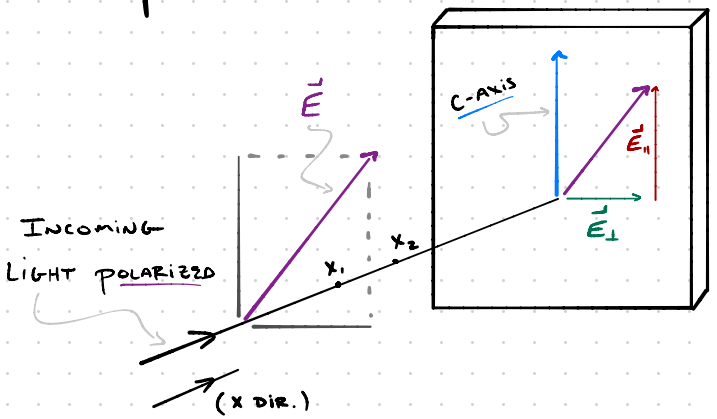


- HOW DOES HALF-WAVE PLATE ROTATE POLARIZATION?

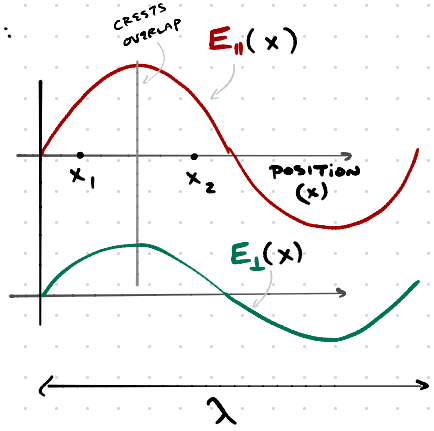
* NOTE THAT THE C-AXIS LIES ∥ TO THE WAVEPLATE SURFACE SO WE DO NOT GET EFFECTS OF DOUBLE REFRACTION.

PHASE RETARDATION

- ELECTRIC FIELD CAN BE DECOMPOSED INTO COMPONENTS $\parallel$ & $\perp$ TO C-AXIS OF WAVEPLATE:



- FOR (LINEARLY) POLARIZED LIGHT IN A NON-BIREFRINGENT ("ISOTROPIC") MATERIAL, THESE COMPONENTS OSCILLATE IN-PHASE:



E.G. @ FIXED INSTANT IN TIME:

- IN BIREFRINGENT MEDIUM, $\vec{E}_{\parallel}$ & $\vec{E}_{\perp}$ OSCILLATE W/ DIFFERENT SPATIAL PERIODICITIES!

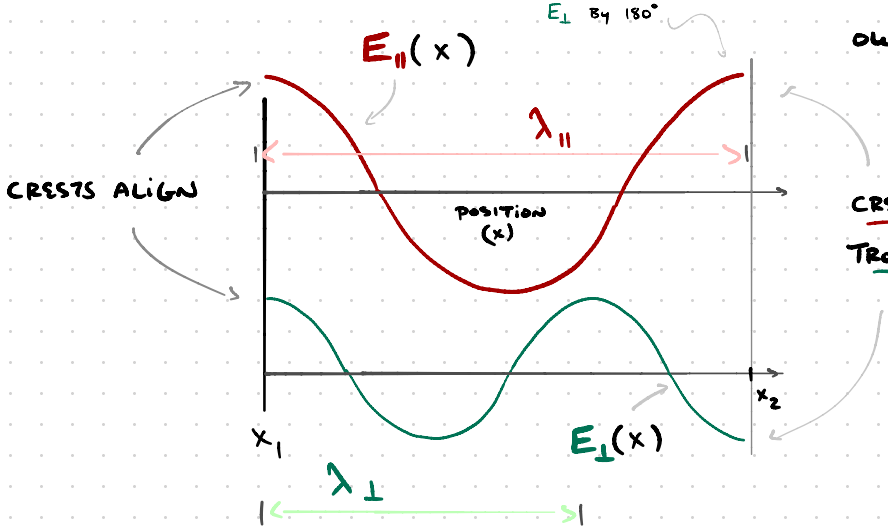
$$\lambda_{\parallel} = \frac{v_{\parallel}}{f} = \frac{c/n_e}{f} = \frac{\lambda_0}{n_e} \quad \left[\lambda_0 \equiv \frac{c}{f} : \text{"VACUUM WAVELENGTH"}\right]$$

(n_e: EXTRAORDINARY)

$$\lambda_{\perp} = \lambda_0/n_o$$ (n_o: ORDINARY)

E.G. @ $x = x_1$, $E_{\parallel}$ & $E_{\perp}$ MIGHT BE IN PHASE...

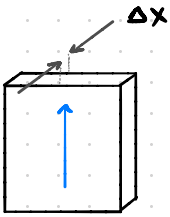
...WHILE A DISTANCE $\Delta x \equiv x_2 - x_1$ DOWNSTREAM, $E_{\parallel}$ & $E_{\perp}$ COULD BE 180° OUT OF PHASE!



- A HALF-WAVE PLATE IS DESIGNED TO HAVE A THICKNESS Δx SO THAT THE TWO WAVES EMERGE FROM THE WAVEPLATE 180°* OUT-OF-PHASE:

$$\frac{\Delta x}{\lambda_{\parallel}} - \frac{\Delta x}{\lambda_{\perp}} = \frac{\Delta x}{\lambda_0}(n_e - n_o) = \frac{1}{2}^{*}$$

($\frac{\Delta x}{\lambda_{\parallel}}$: # OF $\parallel$ WAVES; $\frac{\Delta x}{\lambda_{\perp}}$: # OF $\perp$ WAVES)

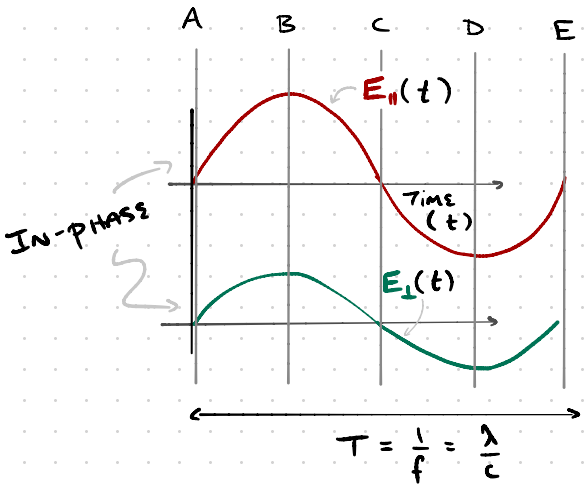


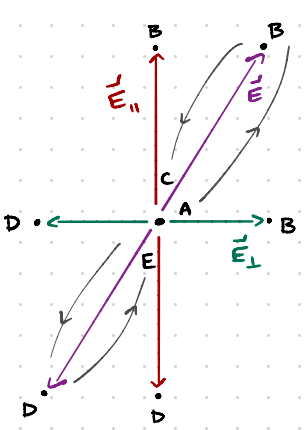
* I.E. ONE "HALF-WAVE" * OR $-\frac{1}{2}$, OR $\pm\frac{3}{2}$, $\pm\frac{5}{2}$, ETC.

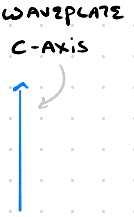
Half-Wave Plate & Rotation of Polarization

- What is the effect of this 180° phase shift?

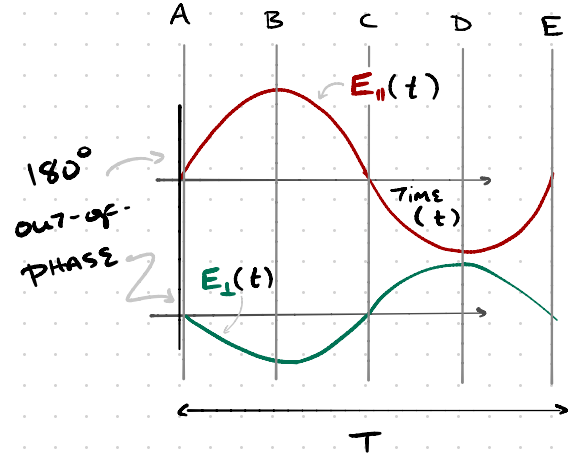
- **Before waveplate:**

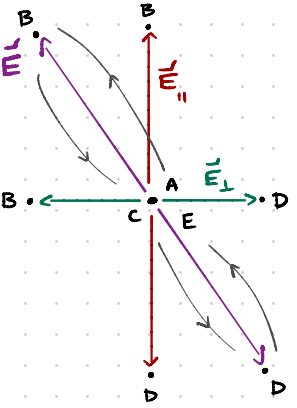


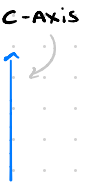




- **After waveplate:**

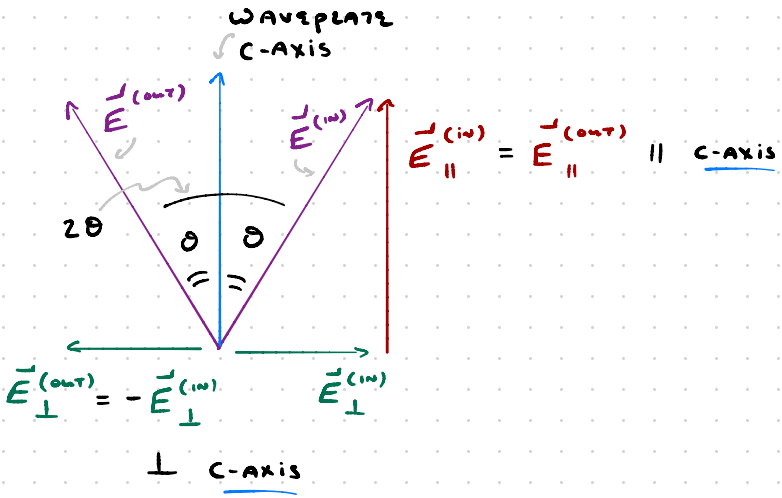






HALF-WAVE PLATE & ROTATION OF POLARIZATION

- WHAT IS THE EFFECT OF THIS 180° PHASE SHIFT?



- POLARIZATION IS REFLECTED ABOUT C-AXIS OF HWP.

- IF $\vec{E}^{(in)}$ MAKES ANGLE θ W/ C-AXIS, THEN POLARIZATION IS ROTATED BY 2θ.

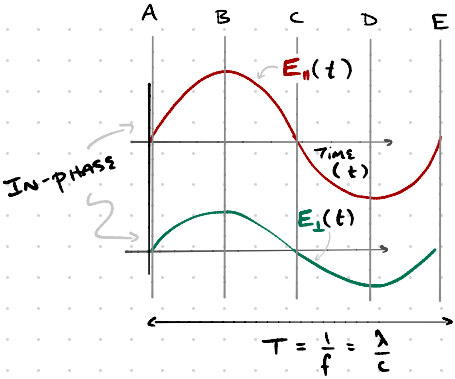
QUARTER-WAVE ($\lambda/4$) PLATE

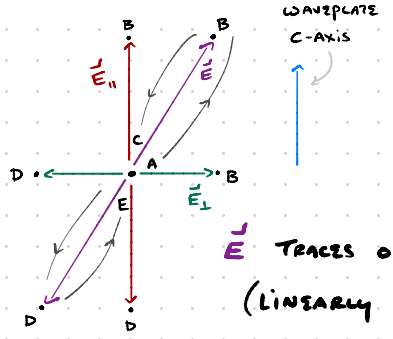
- IF WE REDUCE THICKNESS Δx OF HALF-WAVE PLATE BY FACTOR 2x, WE OBTAIN A QUARTER-WAVE PLATE: (QWP)

$$\left[\frac{\Delta x}{\lambda_0} \left[n_e - n_o \right] = 1/4 \right]$$

- $E_{\parallel}$ WAVE & $E_{\perp}$ WAVE ARE 90° OUT OF PHASE EMERGING FROM A QWP:

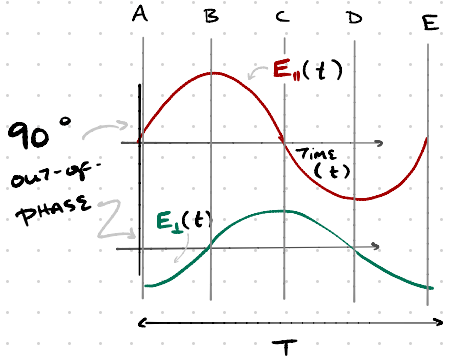
- BEFORE QWP:

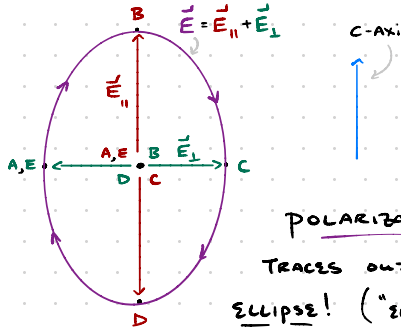




$\vec{E}$ TRACES OUT A LINE (LINEARLY POLARIZED)

- AFTER QWP:

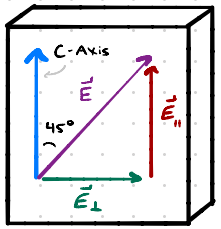




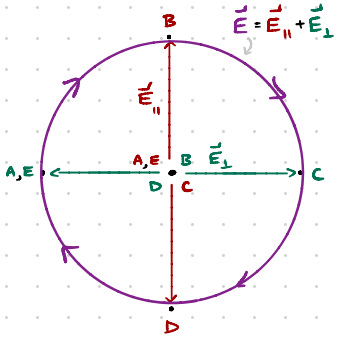
POLARIZATION TRACES OUT AN ELLIPSE! ("ELLIPTICAL POLARIZATION")

Circular Polarization

- If incident light is linearly polarized @ 45° to c-axis of QWP, then $|\vec{E}_{\parallel}| = |\vec{E}_{\perp}|$:
  - output polarization traces a circle* in time!



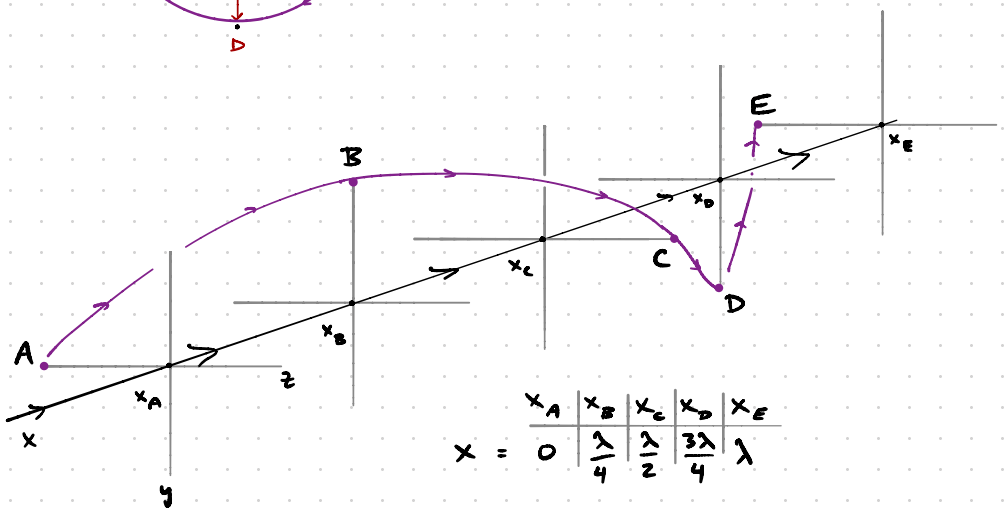
Quarter-wave plate



C-Axis

t	t_A	t_B	t_C	t_D	t_E
	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T

1 period



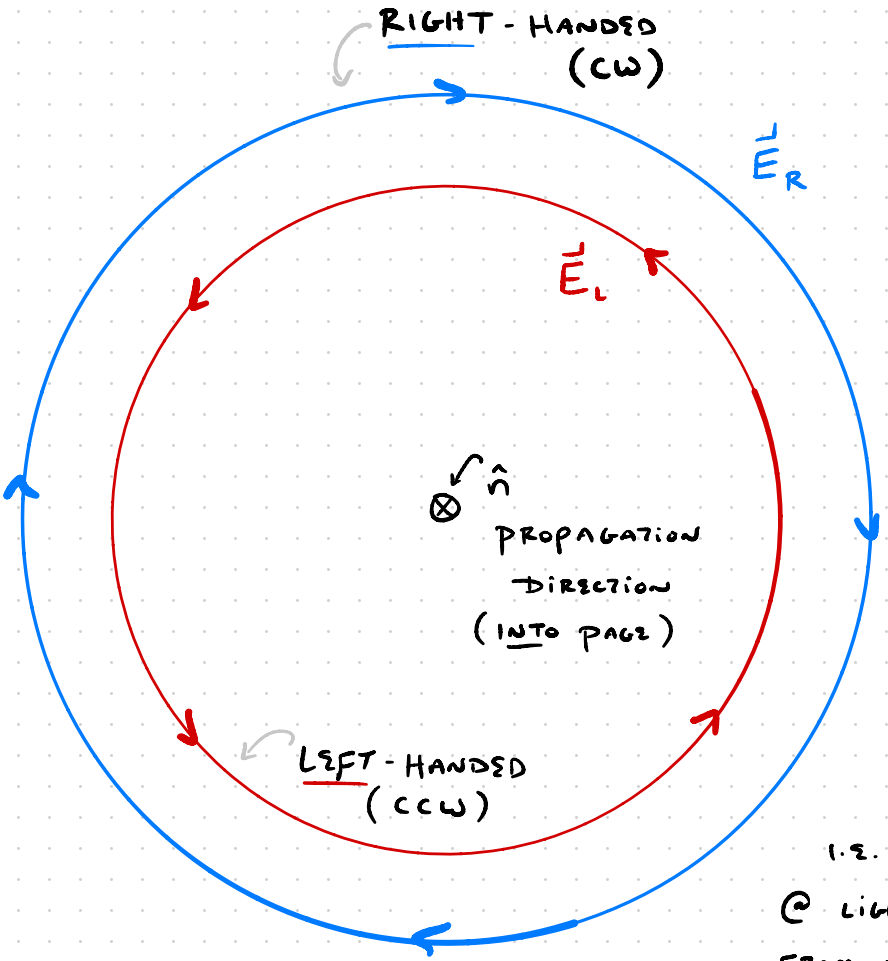
x	x_A	x_B	x_C	x_D	x_E
	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ

Traces out a helix as light moves downstream!

* Elliptical polarization is thus a "mixture" of linear & circular polarizations.

RIGHT - & LEFT - HANDED CIRCULAR POLARIZATION

- CIRCULAR POLARIZATION COMES IN TWO FLAVORS:

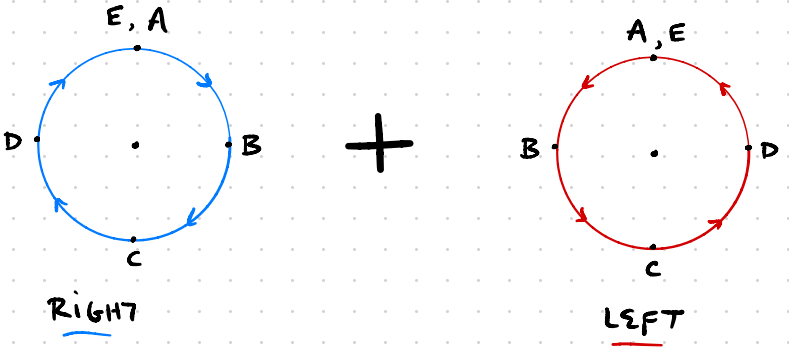


I.E. LOOKING @ LIGHT BEAM FROM BEHIND.

- POINT THUMB TOWARDS $\hat{n}$, FINGERS CURL TOWARDS $\vec{E}$.

RIGHT- & LEFT-HANDED CIRCULAR POLARIZATION

- LINEAR POLARIZATION is AN EQUAL SUPERPOSITION of RIGHT- & LEFT-HANDED CIRCULAR POLARIZATION!



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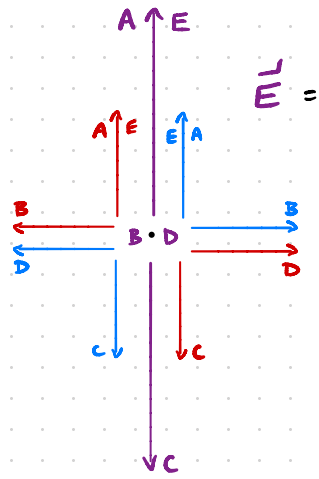


$\vec{E} = \vec{E}_R + \vec{E}_L =$ LINEARLY POLARIZED!

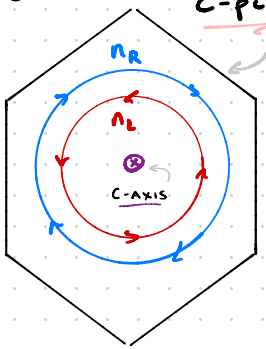
[if $|\vec{E}_R| = |\vec{E}_L|$]

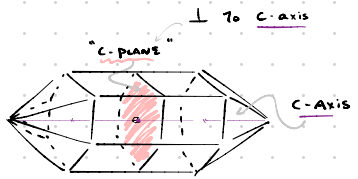
CIRCULAR BIREFRINGENCE ("OPTICAL ACTIVITY")

- INDEX OF REFRACTION CAN DEPEND ON THE "HANDED-NESS" (I.E. RIGHT OR LEFT) OF THE CIRCULAR POLARIZATION!

- E.G. QUARTZ*:

* RECALL FROM EARLIER THAT QUARTZ HAS NO LINEAR BIREFRINGENCE IN THE C-PLANE





$$n_R - n_L \simeq 7 \times 10^{-5}$$

(I.E. LEFT-HANDED POLARIZED LIGHT TRAVELS SLIGHTLY FASTER ALONG THE C-AXIS OF QUARTZ THAN RIGHT-HANDED LIGHT.)

- E.G., @ $\lambda_0 = 600\,nm$, A $\Delta x = 4mm$ THICK QUARTZ PLATE GIVES:

(YELLOW)

$$\frac{\Delta x}{\lambda_0}\left[n_R - n_L\right] \simeq \frac{1}{2},$$

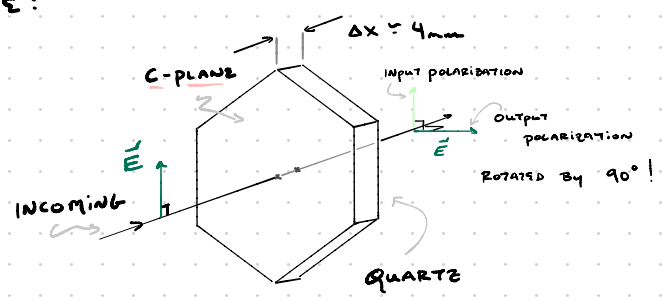
(I.E. A LEFT-HANDED WILL EXIT THE QUARTZ PLATE 180° BEHIND A RIGHT-HANDED WAVE.)

SO THAT WE HAVE WHAT YOU MIGHT CALL A "CIRCULAR HALF-WAVE PLATE".

- WHAT IS THE EFFECT OF THIS 180° PHASE SHIFT ON LINEARLY POLARIZED LIGHT?

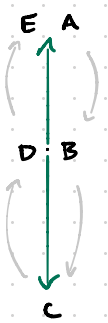
Circular Birefringence ("Optical Activity")

- Send, e.g., vertically polarized light into our quartz plate:



- Before quartz plate:

$\vec{E} = \vec{E}_R + \vec{E}_L$



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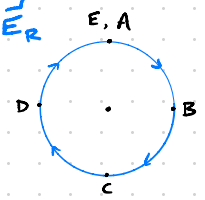
Right + Left components in-phase



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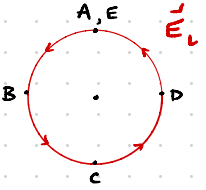
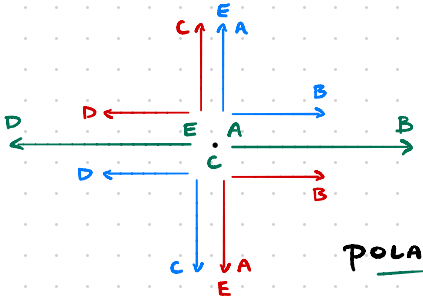


- After quartz plate:

$\vec{E} = \vec{E}_R + \vec{E}_L$



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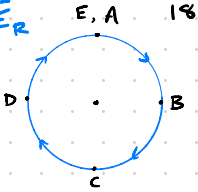
Right + Left components 180° out of phase!



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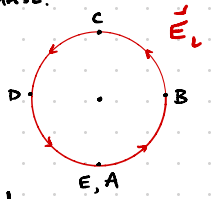


Polarization rotated by 90°!

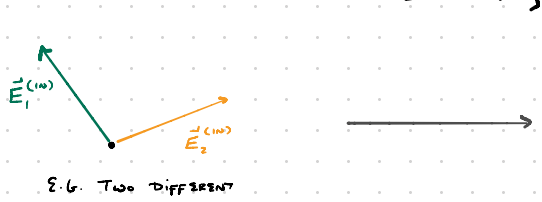
("Optical Activity")

Optical Activity

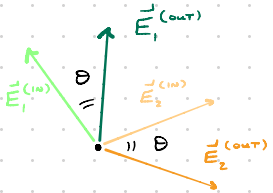
- In general, rotation angle θ is determined by thickness Δx, wavelength λ_0, & circular birefringence $\Delta n_c \equiv n_R - n_L$:

$$\theta = 360^\circ \times \frac{\Delta x}{\lambda_0} \times \Delta n_c$$

- Rotation is <u>independent</u> of <u>direction</u> of input polarization! [contrast w/ HWP described earlier]



E.g. two different input polarizations...



... are rotated by <u>same</u> amount, i.e. θ, by "optically active" medium!

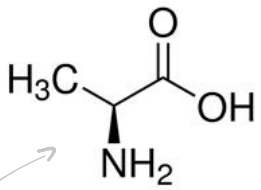
- <u>Terminology</u>: materials acting to rotate polarization clockwise [from perspective facing light head-on] are known as dextrorotary ("right"), w/ levorotary ("left") materials rotating polarizations counter-clockwise.

ORIGIN OF CIRCULAR BIREFRINGENCE

["CHIRALITY"]

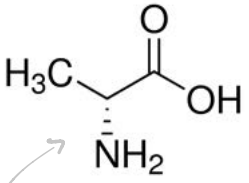
- OPTICALLY ACTIVE MATERIALS ARE "CHIRAL", I.E. THEY ARE NOT THEIR OWN MIRROR IMAGE.

- EXAMPLE: ALANINE α-HELIX



MIRROR IMAGE

"ENANTIOMERS"

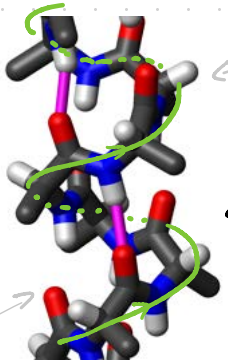


L-ALANINE, MAKES UP ~8% OF PROTEINS

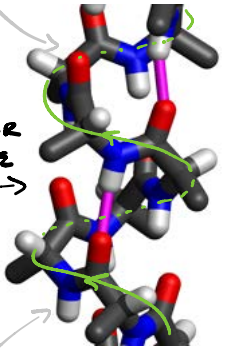
D-ALANINE, GENERALLY ABSENT IN PROTEINS

α-HELIXES

FORMS "RIGHT-HANDED" HELICAL CHAINS



MIRROR IMAGE



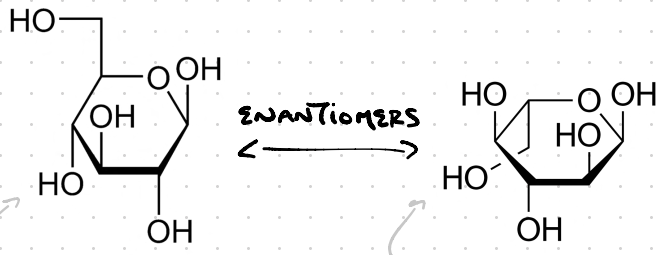
FORMS "LEFT-HANDED" HELICAL CHAINS

DEXTROROTARY

LEVOROTARY

Chirality & Optical Activity (cont.):

- Another example: Glucose



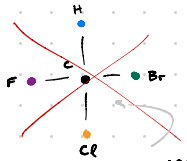
D-Glucose ("Dextrose")

- Synthesized & metabolized by living things.
- Dextrorotary
- Corn syrup contains ≈ 30% D-glucose but no L-glucose. Exhibits pronounced dextrorotation!

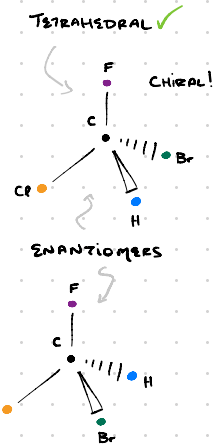
L-Glucose

- Not produced naturally
- Tastes sweet but has no calories*!
- Levorotary

- History of organic chemistry:
  - Existence of dextro- & levo-rotary varieties of substituted methanes [e.g. $CHFClBr$] showed that these molecules were not planar, since planar molecules are their own mirror image [i.e. not chiral]. Led to discovery of tetrahedral bonding in organic molecules [1874]!
    - van't Hoff & Le Bel



planar (2-D) geometry (∅ chiral)



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* But laws of chemistry are mirror symmetric, so D- & L-glucose have same energy!

How can one contain calories but the other not?!