LECTURE 26 NOTES

WAVE OPTICS

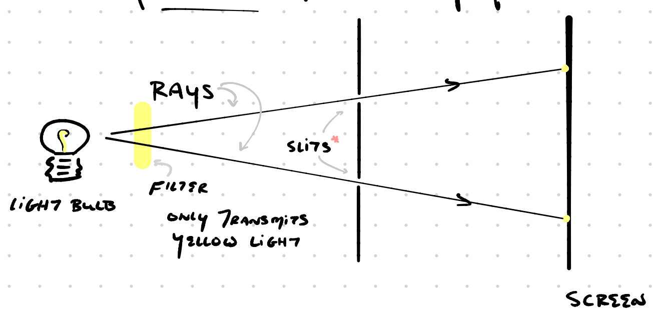
LIMITATIONS OF RAY OPTICS

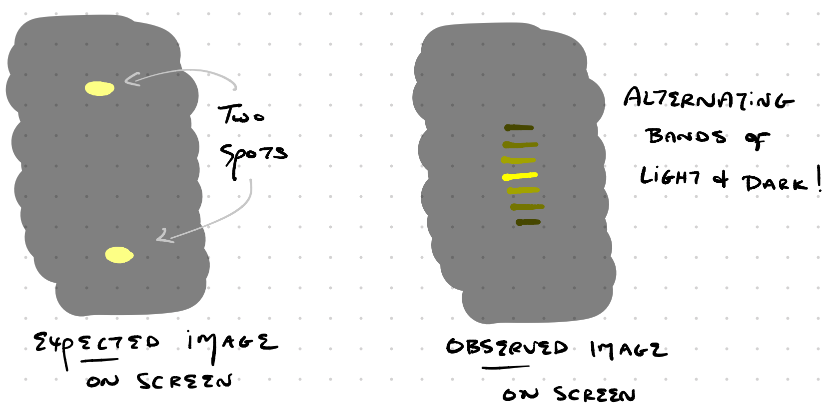
- RAY OPTICS UNABLE TO EXPLAIN EFFECTS OF INTERFERENCE & DIFFRACTION:

- INTERFERENCE:

E.G. TWO SLIT EXPERIMENT [THOMAS YOUNG 1790's]

EXPECTATION FROM RAY OPTICS:



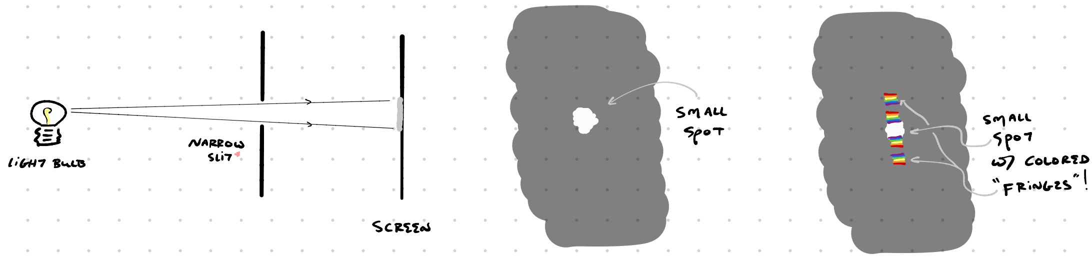


* NOT TO SCALE. ASSUME THE SLITS ARE VERY CLOSE TOGETHER (< 1mm)

Diffraction

- E.G. Single Slit Diffraction
  - Experiment #1: Single Narrow Slit:

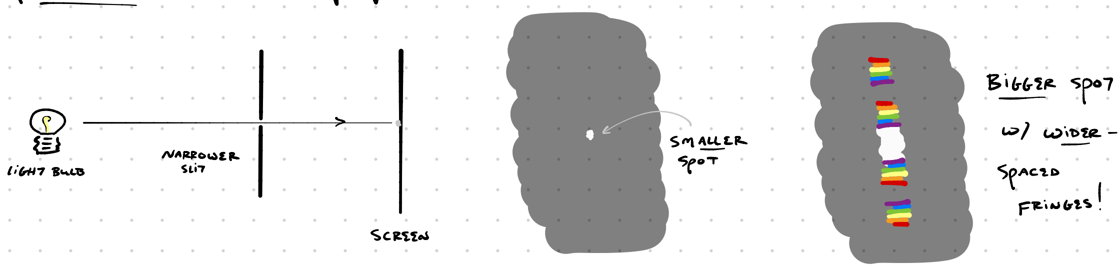
Expectation from Ray optics:



Expected image on screen | Observed image on screen

Experiment #2: Narrower Slit:

Expectation from Ray optics:



Expected image on screen | Observed image on screen

- How do we account for these observations?!

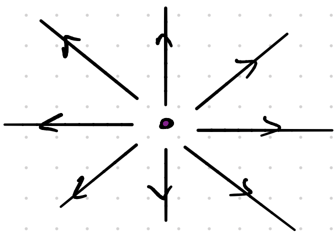
* Narrow means some fraction of a millimeter or so.

WAVE OPTICS

- THE WAVE THEORY OF LIGHT CAN EXPLAIN INTERFERENCE & DIFFRACTION (AS WELL AS REFLECTION + REFRACTION).

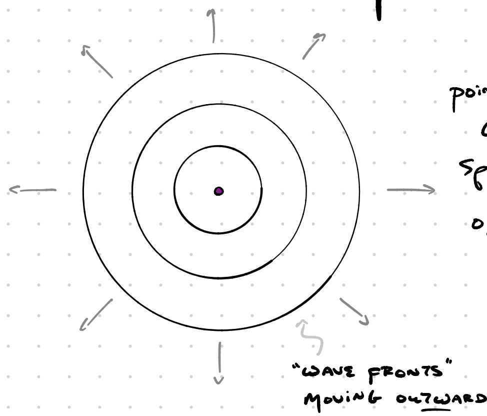
POINT SOURCES

RAY THEORY



EMITS RAYS IN ALL DIRECTIONS

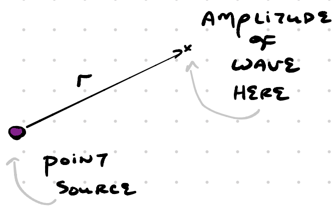
WAVE THEORY



POINT SOURCE GENERATES SPHERICAL WAVES OF LIGHT!

- THE AMPLITUDE $A(r,t)$ OF A WAVE GENERATED BY A POINT SOURCE IS:

$$A(r,t) = \frac{A_0}{r} \cos\left(2\pi\left[\frac{r}{\lambda} - \frac{t}{T}\right]\right)$$



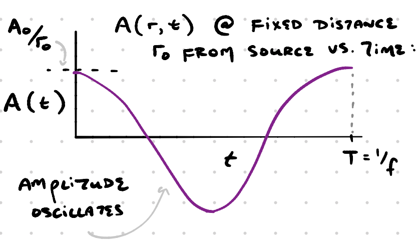
A_0 : STRENGTH OF SOURCE

r : DISTANCE FROM SOURCE

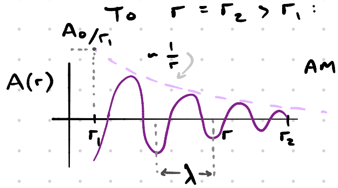
t : TIME

λ / T : WAVELENGTH / PERIOD OF LIGHT EMITTED BY SOURCE $[T = 1/f]$

$A(r,t)$ @ FIXED DISTANCE r_0 FROM SOURCE VS. TIME:



$A(r,t)$ @ FIXED TIME FROM $r = r_1 > 0$ TO $r = r_2 > r_1$:



AMPLITUDE OSCILLATES + DECREASES AS r INCREASES

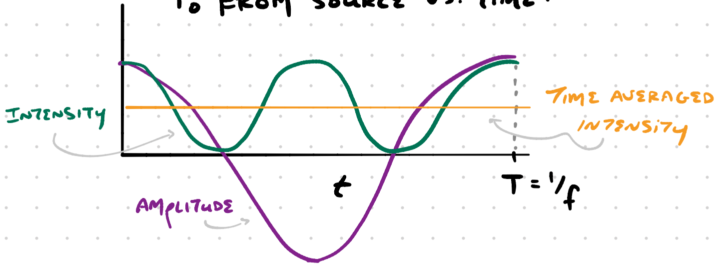
Intensity of wave:

- Intensity $I(r,t)$ of wave is equal to square of amplitude:

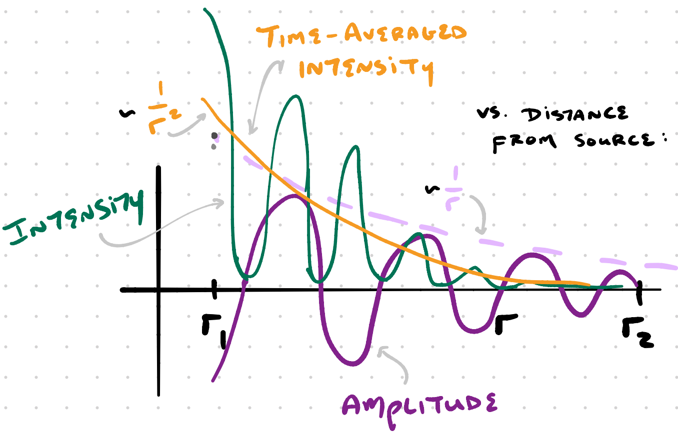
$$I(r,t) = A(r,t)^2$$

(Always positive!)

@ Fixed distance r_0 from source vs. time:

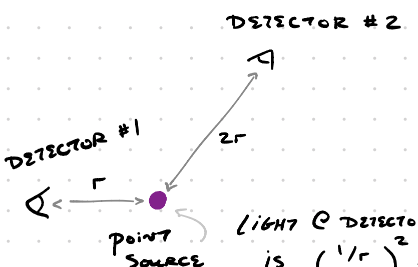


- Frequency of oscillation is very rapid. Typically we can only measure time averaged intensity, which is $\frac{1}{2}$ of peak intensity.



- Time-averaged intensity decreases as inverse square of distance r from source.

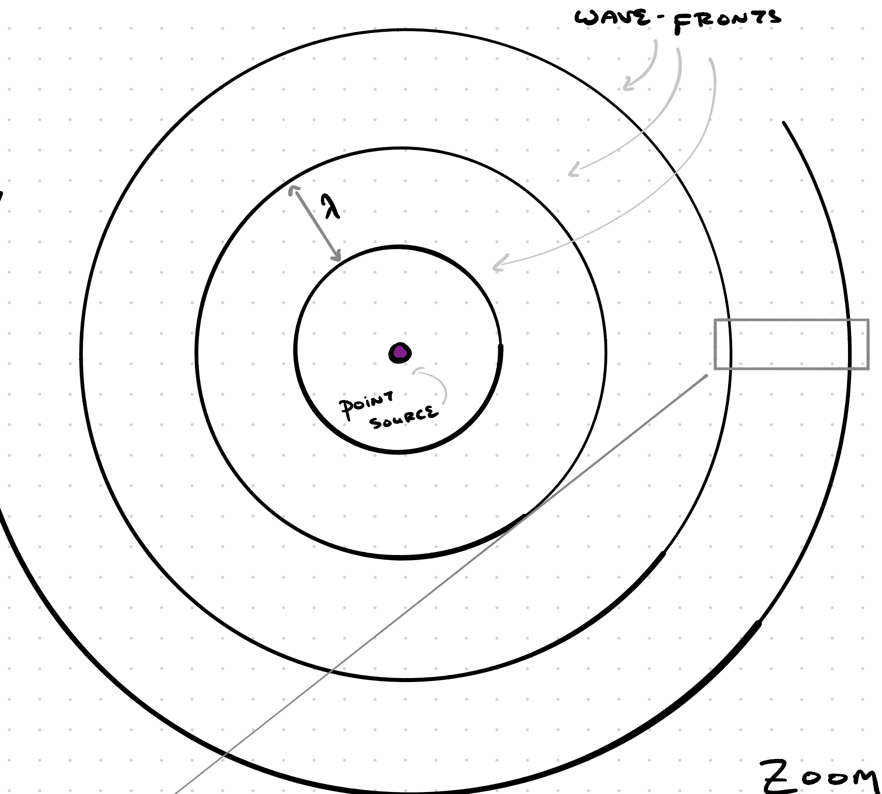
- Energy conservation: Since surface area $A = 4\pi r^2$ of sphere grows as square of distance, power $P = IA$ stays constant as wave moves outwards.



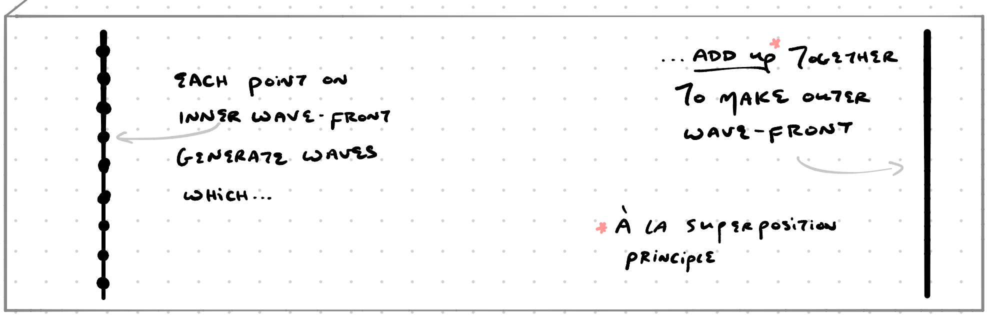
Light @ detector #1 is $\left(\frac{1/r}{1/2r}\right)^2 = 4\times$ brighter!

Huygen's Principle:

- EACH point ON A "WAVE-FRONT" is A SOURCE OF SPHERICAL WAVES:

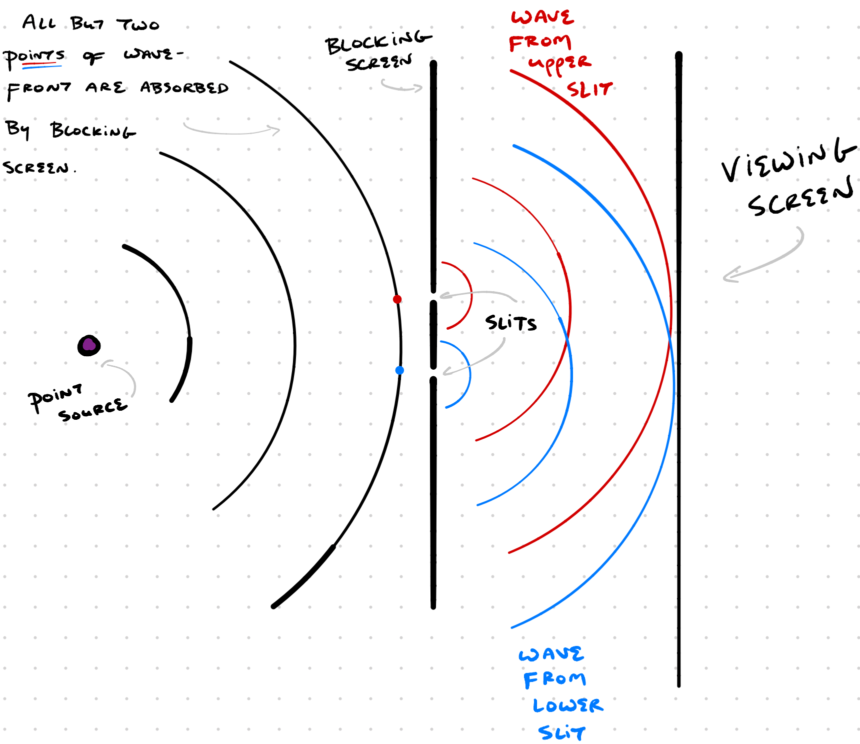


Zoomed in:

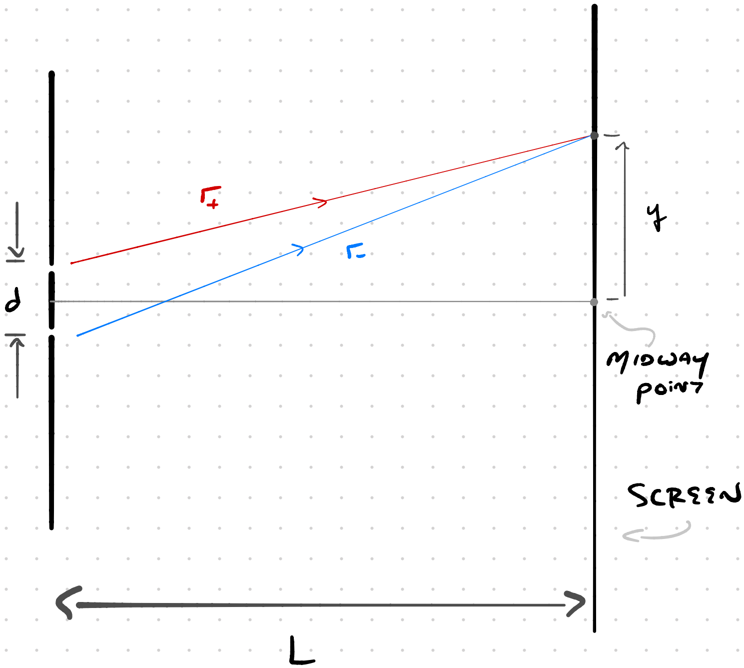


Two Slit Interference:

- If we illuminate two narrow, closely-spaced slits w/ a point source, then, via Huygen's principle, we create two copies of the point source, one @ each slit:



- What is the expected intensity @ the screen?



STRATEGY:

CALCULATE WAVE AMPLITUDE @ SCREEN USING SUPERPOSITION PRINCIPLE, THEN SQUARE TO GET EXPECTED INTENSITY.

AMPLITUDE $A(y,t)$ @ DISTANCE y FROM MIDPOINT BETWEEN SLITS IS:

$$A(y,t) = A_+(r_+,t) + A_-(r_-,t)$$

(WAVE FROM UPPER SLIT; WAVE FROM LOWER SLIT)

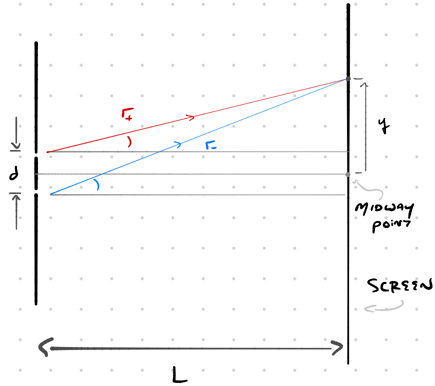
$$A_+(r_+,t) = \frac{A_0}{r_+}\cos\left(2\pi\left[\frac{r_+}{\lambda} - \frac{t}{T}\right]\right)$$ [EQ^N FOR POINT SOURCE]

$$r_+ = \left[\left(y - \left(+\frac{d}{2}\right)\right)^2 + L^2\right]^{1/2}$$ [PYTHAGOREAN THEOREM]

$$A_-(r_-,t) = \frac{A_0}{r_-}\cos\left(2\pi\left[\frac{r_-}{\lambda} - \frac{t}{T}\right]\right)$$

$$r_- = \left[\left(y - \left(-\frac{d}{2}\right)\right)^2 + L^2\right]^{1/2}$$

IF $d \ll L$, THEN



$$\frac{A_0}{r_+} \simeq \frac{A_0}{r_0} \simeq \frac{A_0}{r_-}$$

$$r_0 \equiv \left[y^2 + L^2 \right]^{1/2}$$

SO: $$A(y,t) \simeq \frac{A_0}{r_0} \left[\cos\left(2\pi\left(\frac{r_+}{\lambda} - \frac{t}{T}\right)\right) + \cos\left(2\pi\left(\frac{r_-}{\lambda} - \frac{t}{T}\right)\right) \right]$$

USING TRIG. IDENTITY $\cos a + \cos b = 2\cos\left[\frac{a+b}{2}\right]\cos\left[\frac{a-b}{2}\right]$:

$$A(y,t) \propto \cos\left[\pi\left(\frac{r_+ + r_-}{\lambda} - \frac{t}{T}\right)\right] \cos\left[\pi \frac{r_+ - r_-}{\lambda}\right]$$

(first factor) JUST OSCILLATES RAPIDLY w/ FREQ $f = \frac{1}{T}$, ∅ INTERESTING

(second factor) NO TIME DEPENDENCE!

SO $$I(y) = \frac{1}{2} A_{max}^2(y) \propto \cos^2\left[\pi \frac{r_+ - r_-}{\lambda}\right]$$

$I(y)$: TIME-AVERAGED INTENSITY

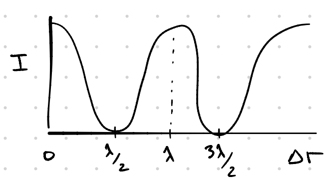
$r_+ - r_-$: "PATH-LENGTH DIFFERENCE" $\equiv \Delta r$

SO WHEN $\Delta r \equiv r_+ - r_-$ IS $0, \pm\lambda, \pm 2\lambda, \ldots$

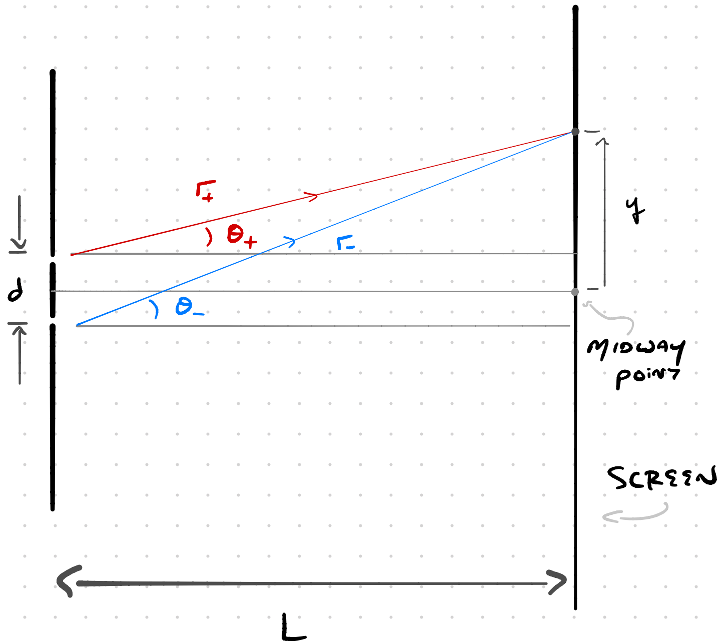
$$\cos^2\left(\pi \frac{\Delta r}{\lambda}\right) = (\pm 1)^2 = +1$$ $I(y)$ IS MAX! "CONSTRUCTIVE INTERFERENCE"

WHILE WHEN $\Delta r = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \ldots$

$$\cos^2\left(\pi \frac{\Delta r}{\lambda}\right) = 0$$ $I(y)$ IS ZERO! "DESTRUCTIVE INTERFERENCE"



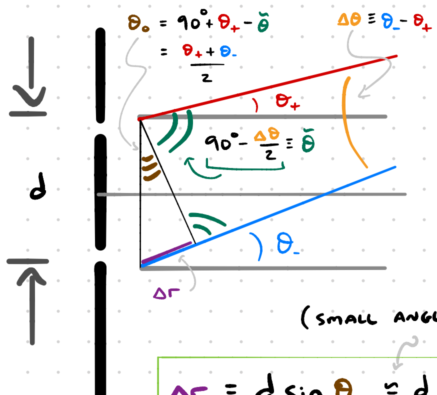
How does Δr depend on y (screen location)?



Two Slit Interference:

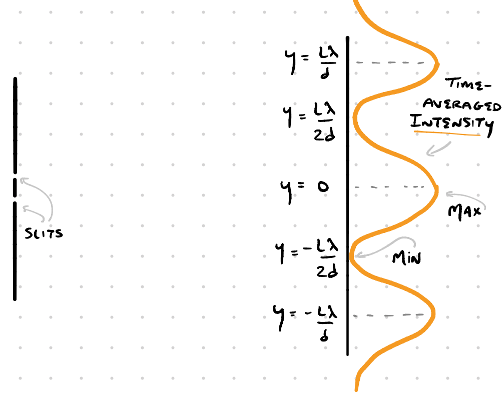
	Constructive	Destructive
Intensity	Maximum	Zero
Path-length difference $\Delta r = r_+ - r_-$	$0, \pm\lambda, \pm 2\lambda \ldots$	$\pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \ldots$
Screen distance y	$0, \pm\frac{L\lambda}{d}, \pm\frac{2L\lambda}{d}$	$\pm\frac{L\lambda}{2d}, \pm\frac{3L\lambda}{2d}, \pm\frac{5L\lambda}{2d}$

Zoomed in:



(small angle approx.)

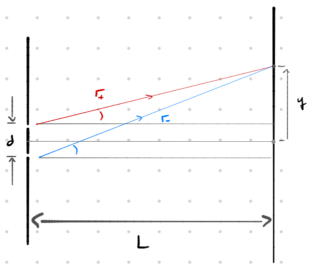
$$\Delta r = d \sin\theta_0 \simeq d\,\frac{y}{L}$$

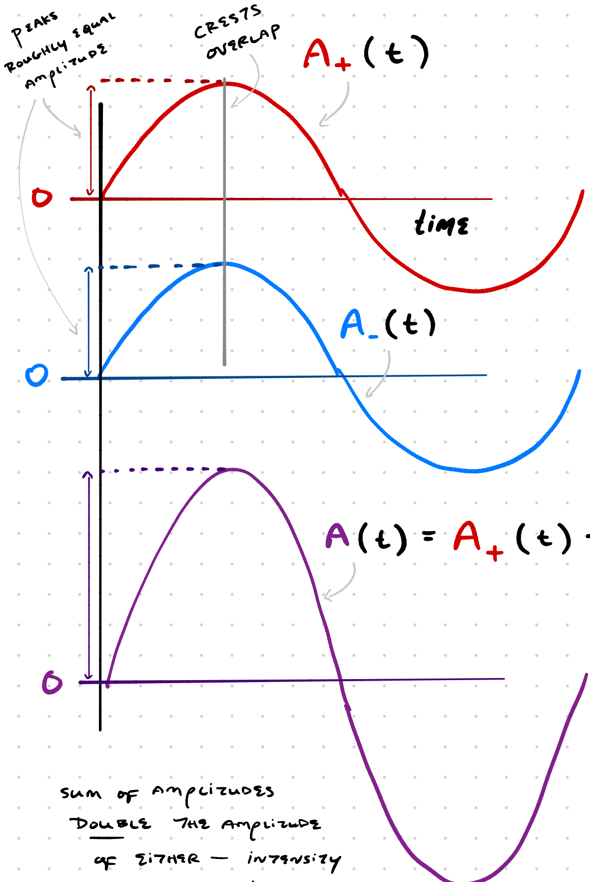


Two-Slit Interference - Conceptual

- @ points on screen where path length difference of two slits is multiple of a wavelength:

i.e. $\Delta r \equiv r_+ - r_- = 0, \pm\lambda, \pm 2\lambda, \ldots$





"Constructive Interference"

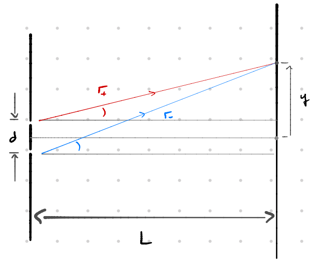
$A(t) = A_+(t) + A_-(t) \simeq 2A_+(t) \simeq 2A_-(t)$

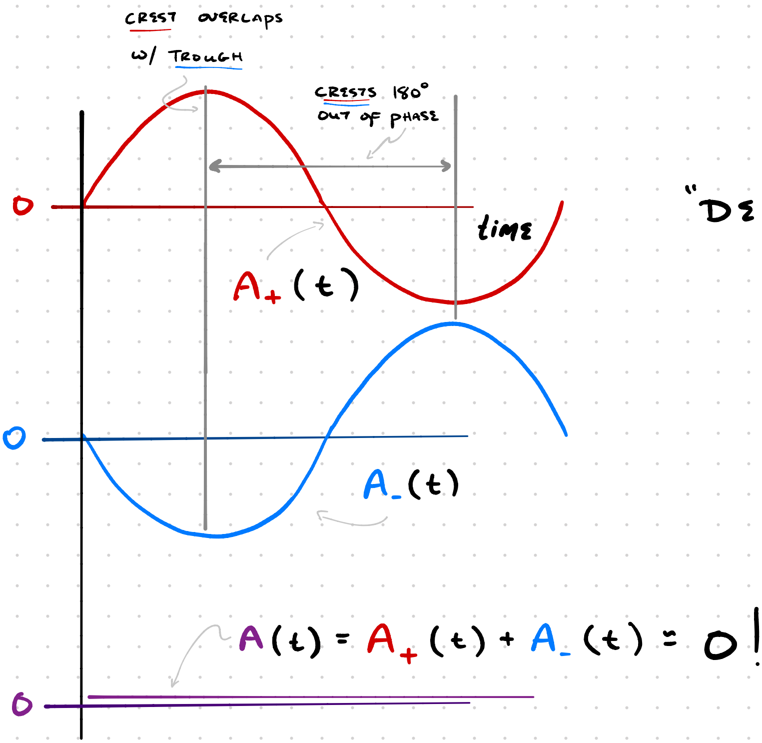
sum of amplitudes double the amplitude of either — intensity quadrupled! ($I = A^2$)

Two-Slit Interference - Conceptual (cont.)

- @ points on screen where path length difference of two slits is odd multiple of a half wavelength:

i.e. $\Delta r \equiv r_+ - r_- = \pm\frac{\lambda}{2}, \pm 3\frac{\lambda}{2}, \pm 5\frac{\lambda}{2}, \dots$

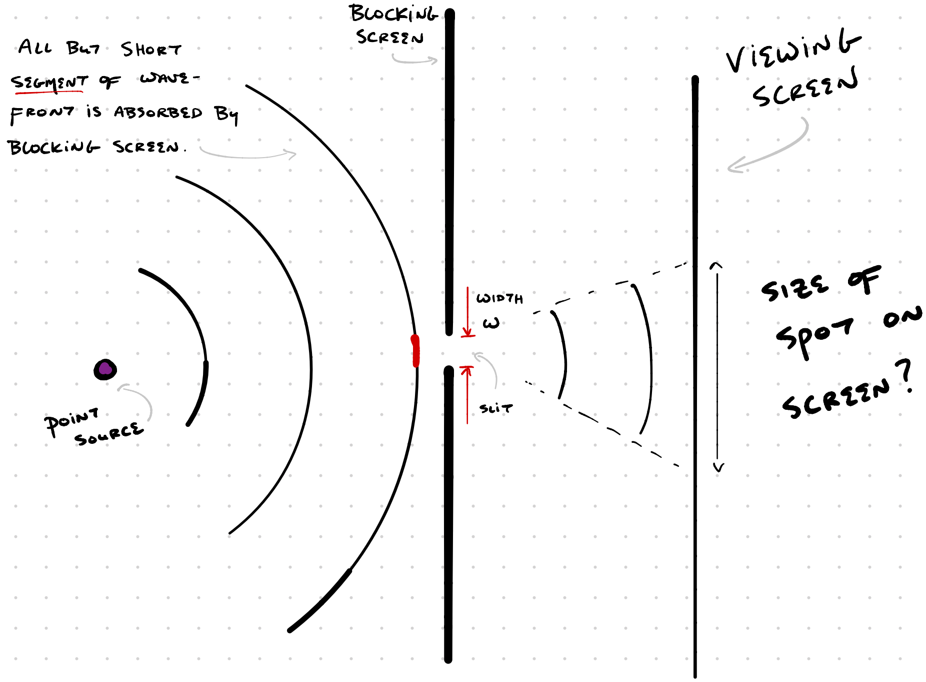




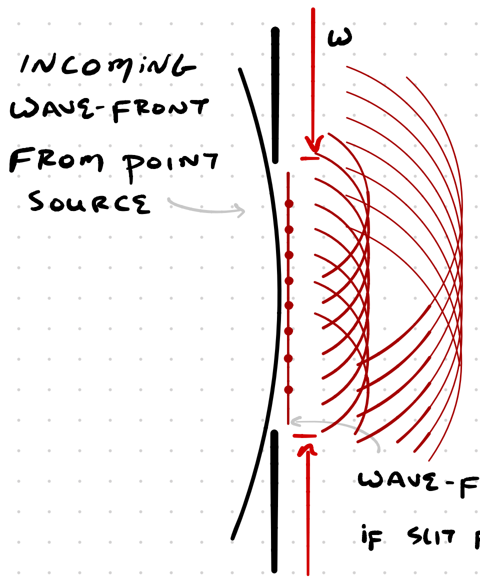
"Destructive interference"

Intensity @ screen is zero @ all times!

Single-Slit Diffraction:



Zoomed in on slit:

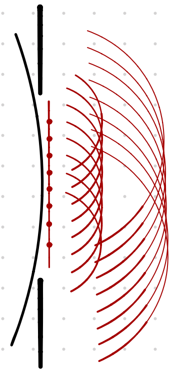


Every point • @ slit becomes a point source of spherical waves, all identical. [Huygen's principle]

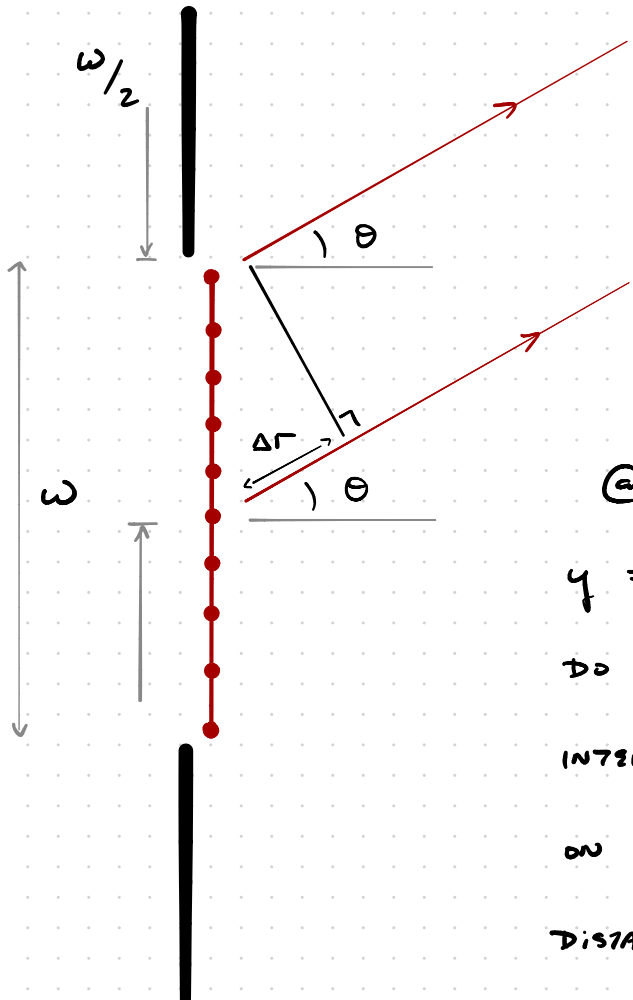
Wave-front approx. straight if slit far from point source

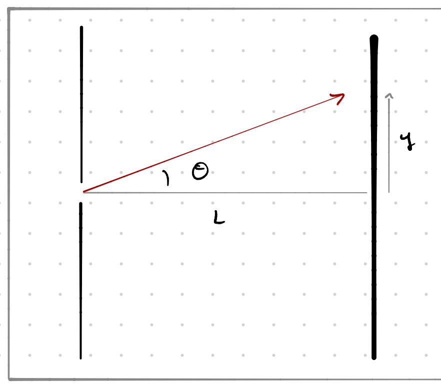
How do these point sources interfere @ the viewing screen?

Single-Slit Diffraction (cont.)



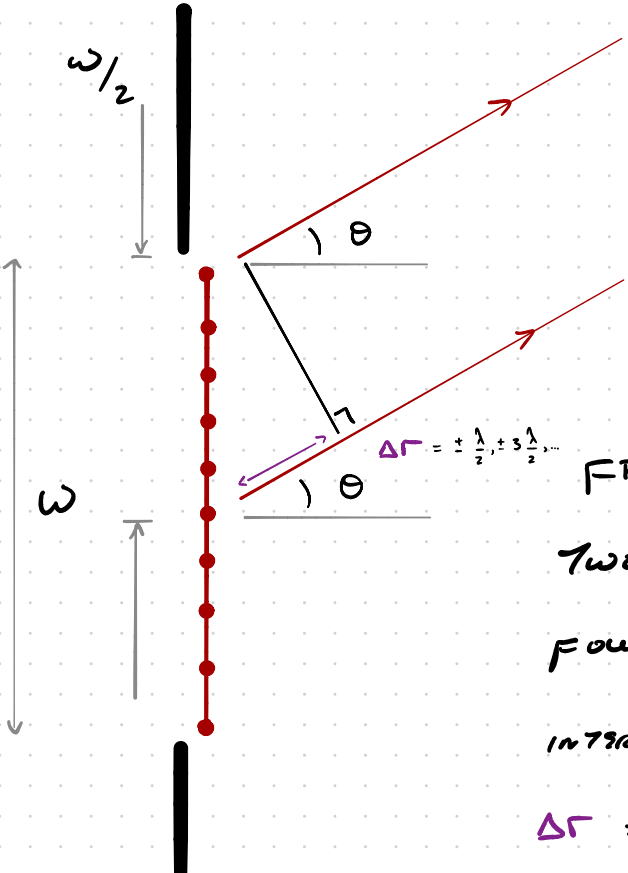
- Infinite # of point sources interfering!
  - Difficult to analyze completely.
- One cool trick* for determining where the points of destructive interference occur:

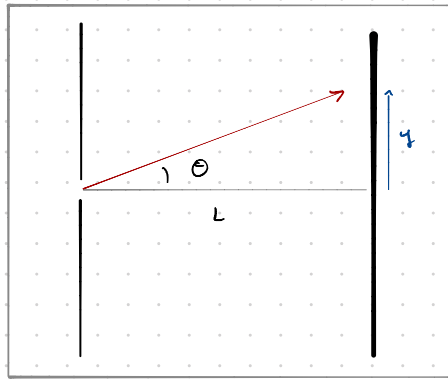




@ what distance $y = L \tan\theta$ along screen do we have destructive interference between points on slit separated by distance $w/2$ [i.e. half the slit width w]?

* Physicists hate him!





From our analysis of two slit diffraction, we found that destructive interference occurs when

$$\Delta r = \pm\frac{\lambda}{2}, \pm 3\frac{\lambda}{2}, \ldots \quad \text{or}$$

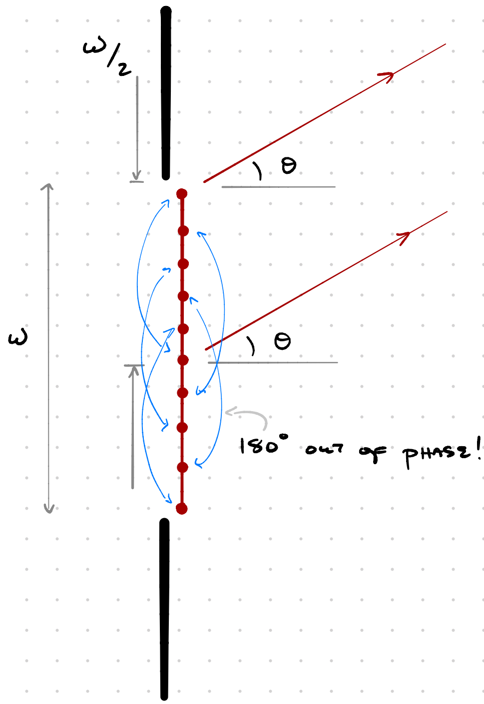
[Valid only when small angle approx $y/L = \tan\theta \simeq \sin\theta \simeq \theta$ holds]

$$y = \pm\frac{L\lambda}{2d}, \pm 3\frac{L\lambda}{2d}, \ldots$$

where d is separation between point sources, which for us now is $w/2$, so:

$$\boxed{y = \pm\frac{L\lambda}{w}}$$

[1^{st} destructive minima, single slit diffraction]

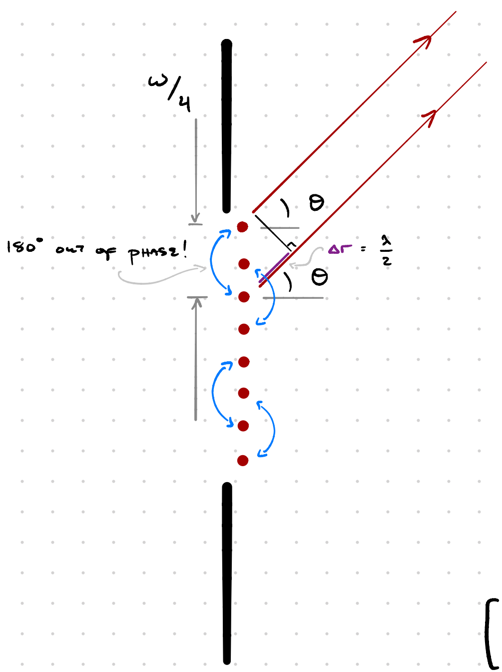


@ THESE ANGLES

$$\frac{y}{L} = \tan\theta = \pm\frac{\lambda}{w},$$

INTENSITY @ SCREEN is ZERO:

LIGHT FROM EVERY POINT ON SLIT CANCELS W/ LIGHT FROM A POINT w/2 AWAY!



ADDITIONAL DESTRUCTIVE MINIMA:

@ LARGER ANGLES, WE GET CANCELLATION BETWEEN POINTS SEPARATED BY w/4:

$$y = \pm 2\frac{L\lambda}{w}$$

[2ND DESTRUCTIVE MINIMA, SINGLE SLIT DIFFRACTION]

ADDITIONAL DESTRUCTIVE MINIMA (CONT.)

- **ANY** EVEN DIVISION OF THE SLIT

$$\left(d = \frac{w}{2},\ \frac{w}{4},\ \frac{w}{6},\ \ldots \right)$$

($\frac{w}{2}$: 1ST DEST. MINIMA; $\frac{w}{4}$: 2ND DEST. MINIMA; $\frac{w}{6}$: 3RD; …: 4TH, 5TH, ETC.)

YIELDS A PAIR OF ANGLES $\theta = \frac{y}{L}$ (SMALL ANGLE APPROX.)

OF "TOTALLY DESTRUCTIVE INTERFERENCE",

I.E. ZERO INTENSITY @ SCREEN:

$$y = \pm\frac{L\lambda}{w},\ \pm 2\frac{L\lambda}{w},\ \pm 3\frac{L\lambda}{w},\ \ldots$$

DESTRUCTIVE MINIMA, SINGLE SLIT DIFFRACTION

SINGLE SLIT DIFFRACTION PATTERN: