

NOTE: THIS EXAMPLE IS JUST
TO GIVE YOU A

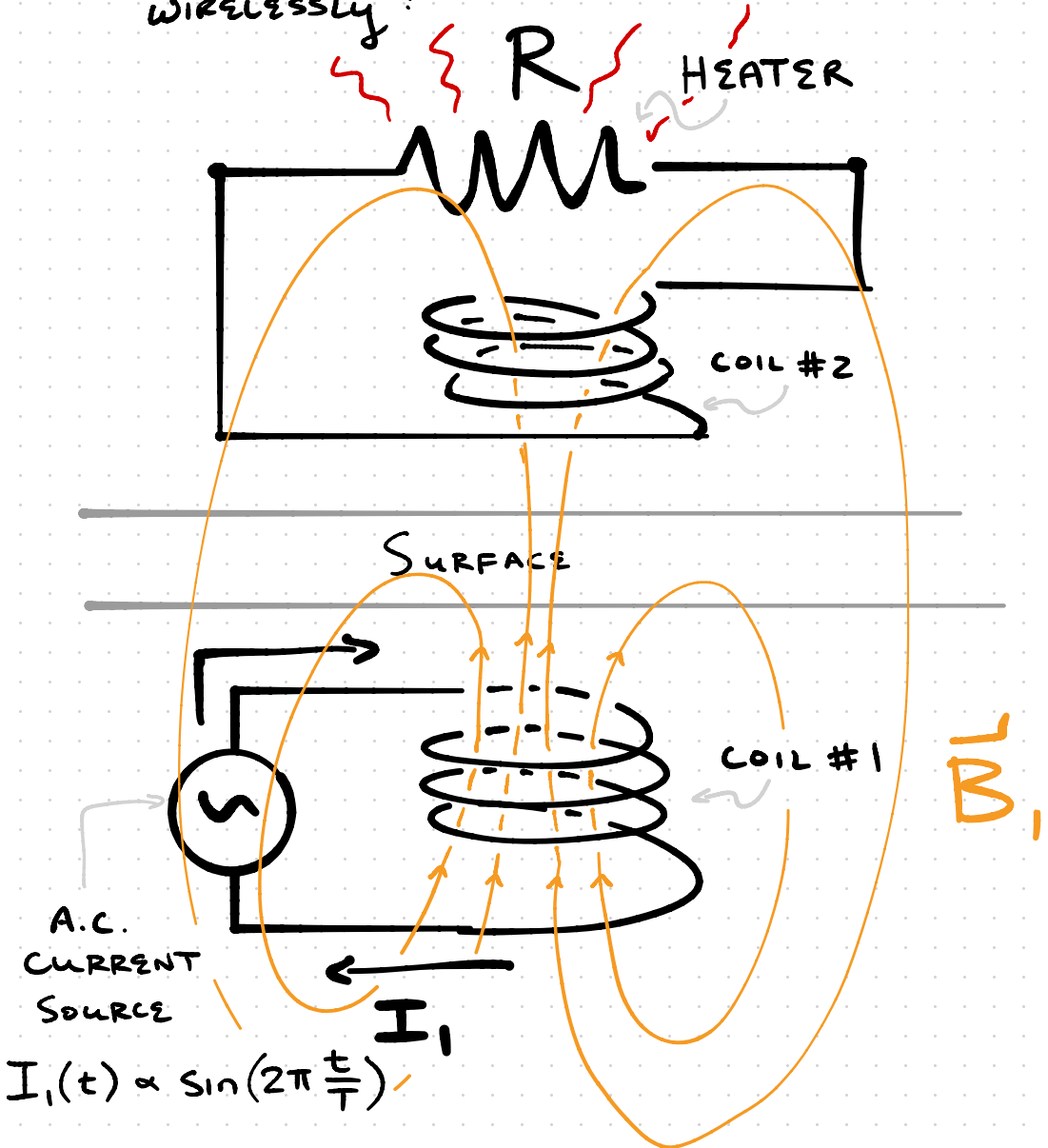
HANDS-ON EXAMPLE OF MUTUAL
& SELF INDUCTANCE. THE ANALYSIS
IS NOT TOO HARD TO FOLLOW BUT
IT WOULD BE DIFFICULT TO SOLVE
A SIMILAR PROBLEM ON YOUR OWN.

DON'T WORRY ABOUT HAVING TO SOLVE
THIS PARTICULAR PROBLEM ON AN EXAM.

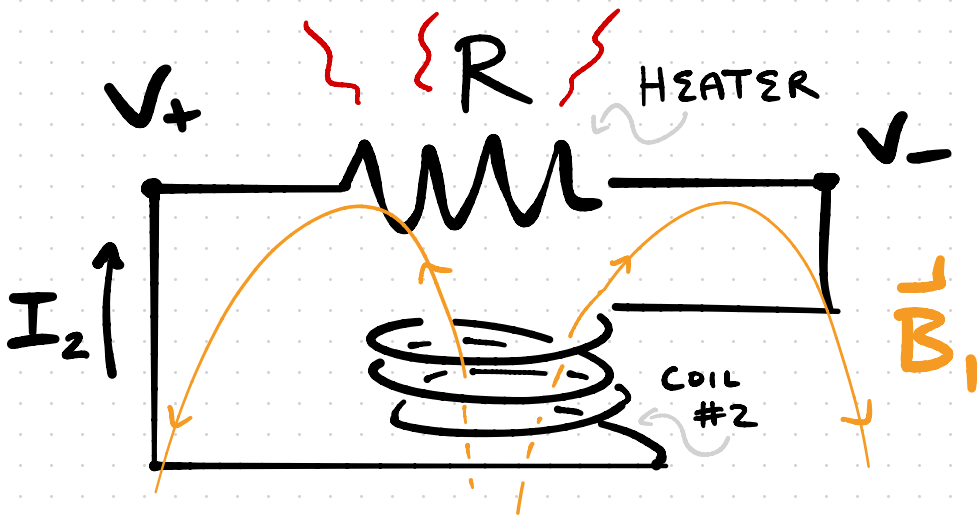
JUST TRY TO UNDERSTAND CONCEPTUALLY.

EXAMPLE: INDUCTION STOVE

- USING A PAIR OF COILS WE CAN GENERATE HEAT WIRELESSLY:



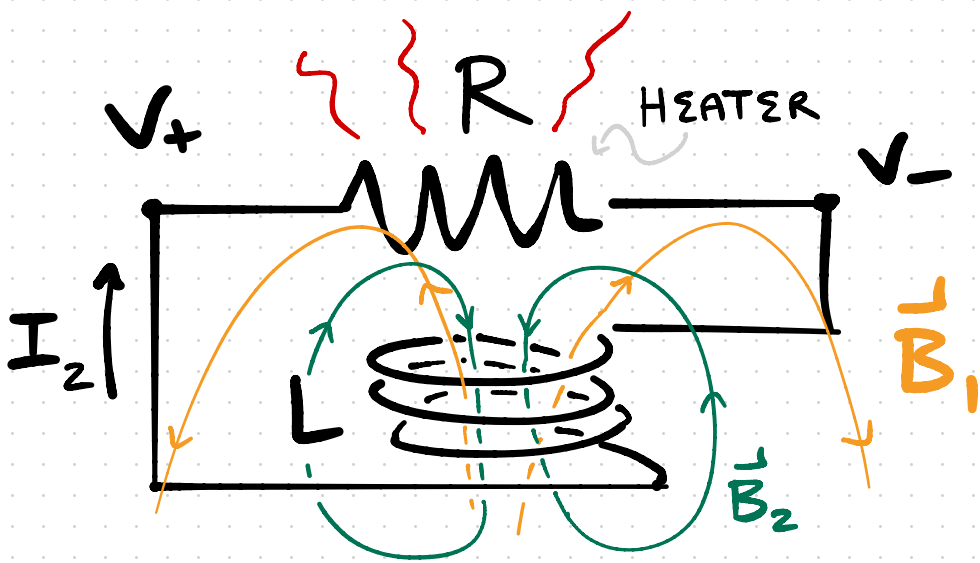
$$I_1(t) \propto \sin\left(2\pi \frac{t}{T}\right)$$



- MUTUAL INDUCTANCE (M) BETWEEN TWO COILS LEADS TO VOLTAGE ACROSS COIL FROM A.C. CURRENT $I_1(t)$:

$$\begin{aligned}
 V &= V_+ - V_- = M \frac{\Delta I_1}{\Delta t} \\
 &= M \frac{2\pi}{T} I_1 \quad \text{(A.C. AMPLITUDE)} \\
 &\quad \left(T = \frac{1}{f} \text{ IS PERIOD OF OSCILLATION} \right)
 \end{aligned}$$

- VOLTAGE DRIVES CURRENT $I_2 = V/R$ IN HEATER.
- SINCE $P = I_2 \times V$ IN RESISTOR, SHOULD WE MAKE R SMALL TO MAXIMIZE POWER TO HEATER?



- CAREFUL! COIL #2 ALSO HAS SELF-INDUCTANCE (L), i.e. THE A.C. CURRENT $I_2(t)$ INDUCED BY $I_1(t)$ WILL GENERATE AN OPPOSING FLUX VIA \vec{B}_2 :

$$\begin{aligned}
 V &= M \frac{\Delta I_1}{\Delta t} - L \frac{\Delta I_2}{\Delta t} \\
 &= M \frac{2\pi}{T} I_1 - L \frac{2\pi}{T} I_2
 \end{aligned}$$

$$\longrightarrow M I_1 = \frac{T}{2\pi} V + L I_2$$

$$\longrightarrow M I_1 = \frac{T}{2\pi} V + L I_2$$

ANALOGY:

MAXIMIZE AREA
ENCLOSED BY FENCE OF
FIXED PERIMETER:

$$a \quad b \quad a + b = \text{fixed}$$

→ MAKE IT A SQUARE:
a = b MAXIMIZES AREA



SUM FIXED,
WANT TO MAXIMIZE
PRODUCT $\propto \sqrt{I_2} = P$

POWER

• HEATER POWER OPTIMIZED WHEN

$$\frac{T}{2\pi} V = L I_2$$

(REARRANGING)

$$\longrightarrow \frac{L}{\sqrt{I_2}} = \frac{L}{R} = \frac{T}{2\pi}$$

UNITS OF TIME!

- SO WE WANT TO PICK OUR HEATER RESISTANCE R SO THAT THE "TIME CONSTANT" $\tau \equiv \frac{L}{R}$ OF OUR HEATER + COIL COMBO MATCHES THE PERIOD OF OSCILLATION T OF THE CURRENT $I_1(t)$ SUPPLYING THE POWER!

* W/IN A FACTOR OF 2π ☺