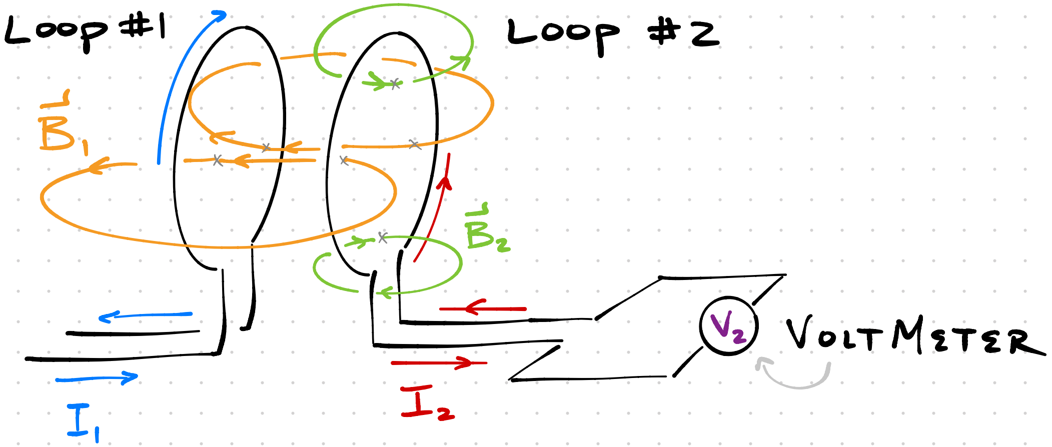
LECTURE 22 NOTES

INDUCTORS + RL CIRCUITS

- RECAP:
  - ELECTROMAGNETIC INDUCTION:



- CHAIN OF INDUCTION:

CHANGING I_1 → CHANGING $\vec{B}_1$ → CHANGING Φ_2 → emf in Loop #2 *→ I_2 *→ V_2

* SEE LECTURE 20 NOTES FOR DETAILS.

- EACH "EFFECT" IN THE CHAIN IS PROPORTIONAL TO THE "CAUSE" PRECEDING IT*:

$$I_1 \propto \vec{B}_1 \propto \Phi_2 \propto \frac{\Delta \Phi_2}{\Delta t} \propto \mathcal{E}mf_2 = V_2$$

- IN PARTICULAR:

$$\frac{\Delta I_1}{\Delta t} \propto V_2$$

"A CHANGE IN THE CURRENT THRU LOOP #1 INDUCES A VOLTAGE ACROSS LOOP #2."

- THE CONSTANT FOR THIS PROPORTIONALITY IS CALLED THE "MUTUAL INDUCTANCE" (M) OF LOOPS #1 & #2:

$$V_2 = M \frac{\Delta I_1}{\Delta t}$$

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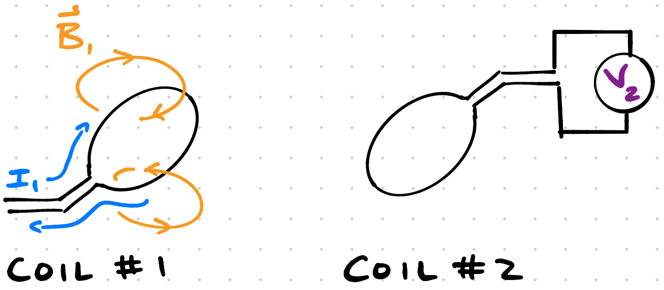
* THIS IS A RESULT OF THE SUPERPOSITION PRINCIPLE FOR ELEC. & MAG. FIELDS.

• MUTUAL INDUCTANCE

- M MEASURES HOW "RESPONSIVE" ONE COIL IS IN CHANGES IN THE OTHER:

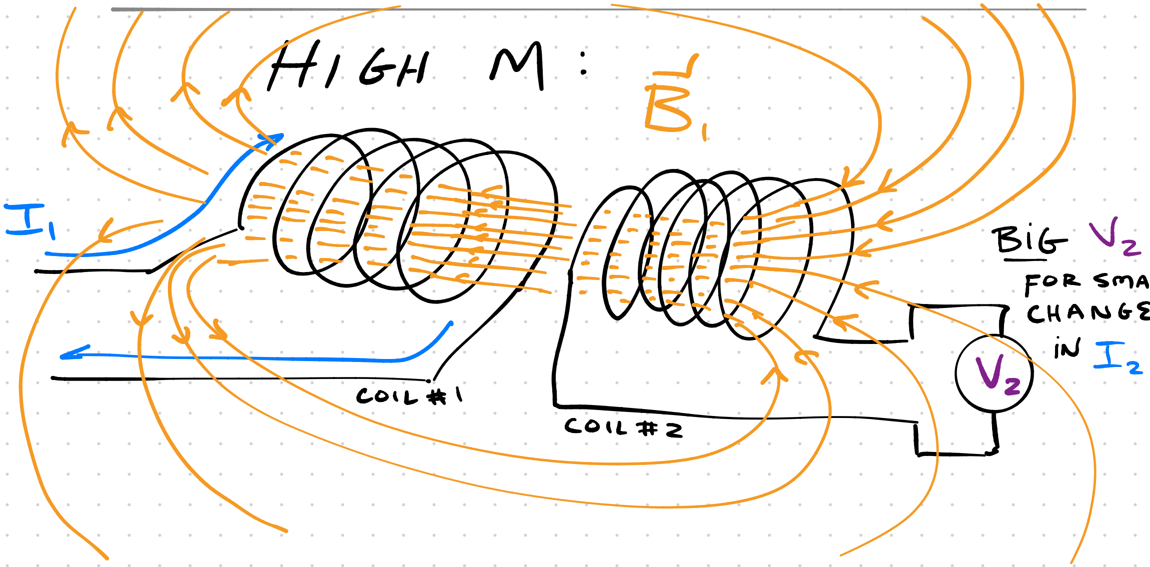
LOW M:



COIL #2 CAN'T "DETECT" CHANGES IN I_1

$(V_2 \simeq 0)$

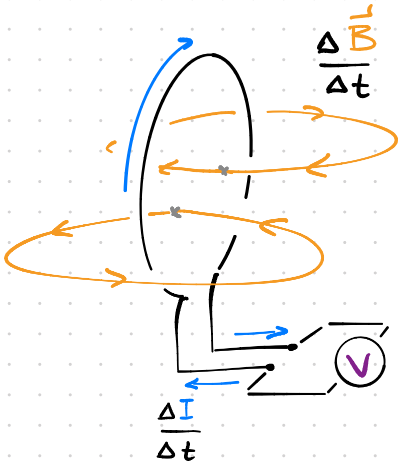
HIGH M:



BIG V_2 FOR SMALL CHANGE IN I_2

SELF - INDUCTANCE

- THIS "CHAIN OF INDUCTION" CAN OCCUR W/IN A SINGLE LOOP / COIL !



$$\frac{\Delta I}{\Delta t} \longrightarrow \frac{\Delta \vec{B}}{\Delta t} \longrightarrow \frac{\Delta \Phi}{\Delta t} \longrightarrow \mathcal{E}mf = V$$

- SO A LOOP / COIL HAS A "SELF - INDUCTANCE"* (L) SUCH THAT:

$$V = L \frac{\Delta I}{\Delta t}$$

- S.I. UNIT OF INDUCTANCE IS THE HENRY (H)

$$1 \, H = 1 \, \frac{V \times s}{A}$$

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* OR JUST "INDUCTANCE" FOR SHORT

- INDUCTANCES IN GENERAL DIFFICULT TO CALCULATE, BUT FOR A LONG, TIGHTLY-WOUND COIL (A.K.A. A "SOLENOID") WE HAVE:

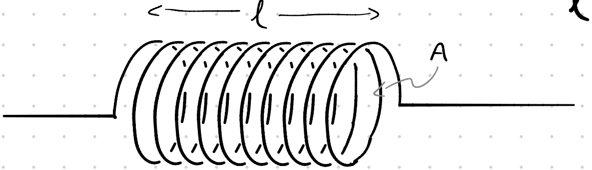
$$L = \frac{\mu_0 N^2 A}{l}$$

$\mu_0 = 1.257 \cdot 10^{-6}$ H/m

N: # LOOPS

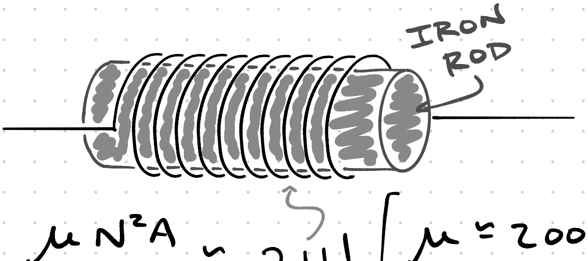
A: LOOP AREA

l: COIL LENGTH



SOLENOID ($L \simeq 1\,\mu$H IF TO SCALE)

- BY PUTTING IRON ROD INSIDE SOLENOID, CAN INCREASE INDUCTANCE BY $\simeq$ 200,000× !



$$L = \frac{\mu N^2 A}{l} \simeq .2\text{H}! \left[\begin{array}{c} \mu \simeq 200{,}000 \times \mu_0 \\ \text{FOR IRON} \end{array}\right]$$

INDUCTORS

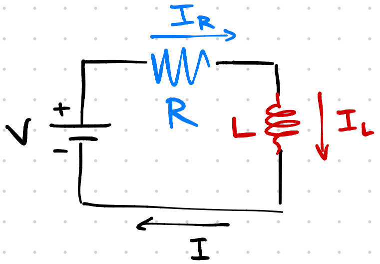
- COILS W/ SIGNIFICANT INDUCTANCE.
- CIRCUIT DIAGRAM SYMBOL: 

- "OPPOSES SUDDEN CHANGES IN CURRENT."

RL CIRCUITS

- CONCEPTUALLY SIMILAR TO R.C. CIRCUITS.

— RL SERIES CIRCUIT



- SUPPOSE @ $t=0$, $I=0$.
- WHAT HAPPENS AS TIME GOES ON $(t>0)$?

IN GENERAL:

- KCL: $I_R = I_L \equiv I$
- KVL: $V = V_R + V_L$

Ⓐ $$\boxed{V = IR + L\frac{\Delta I}{\Delta t}}$$

@ $t=0$

- $I = 0$
- Ⓐ $V = \cancelto{0}{I}R + L\frac{\Delta I}{\Delta t}$

$$= L\frac{\Delta I}{\Delta t}$$

$$\longrightarrow \boxed{\frac{\Delta I}{\Delta t} = V/L}$$ Ⓑ

AS $t \longrightarrow \infty$:

- STEADY STATE
  - NO CHANGING VOLTAGES/CURRENTS

$$\longrightarrow \frac{\Delta I}{\Delta t} = 0$$

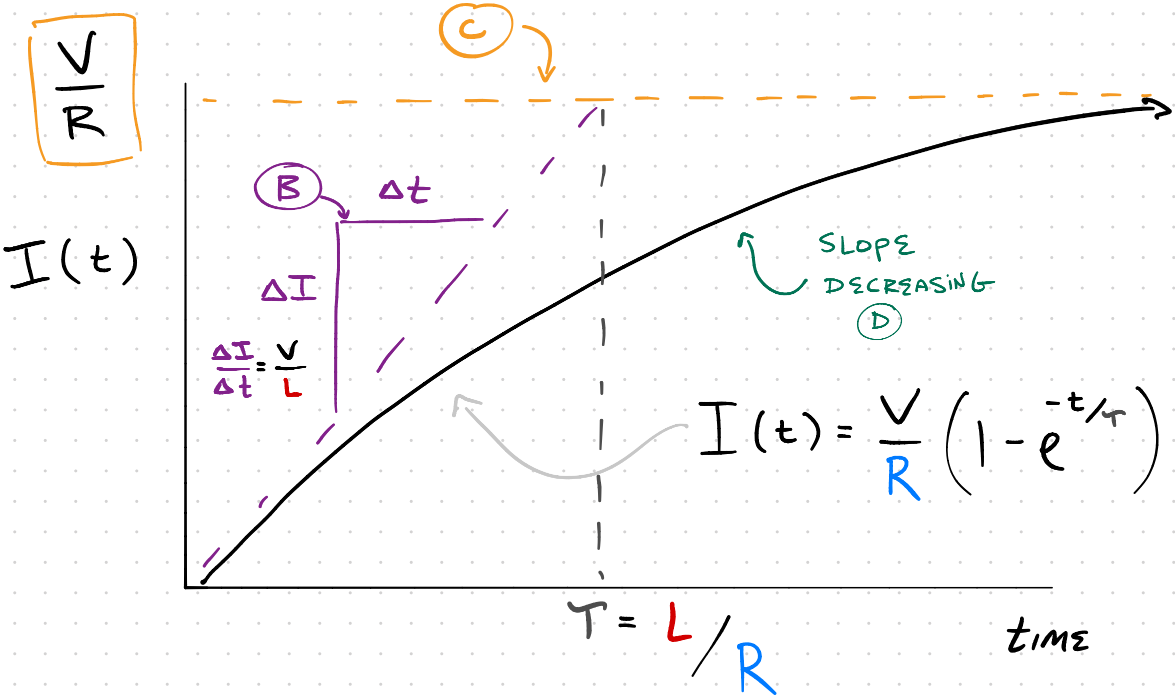
- Ⓐ $V = IR + L \cancelto{0}{\frac{\Delta I}{\Delta t}}$

$$\longrightarrow \boxed{I = \frac{V}{R}}$$ Ⓒ

IN BETWEEN

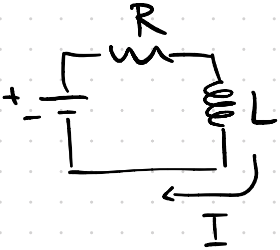
- AS CURRENT BUILDS UP IN CIRCUIT:
- V_R INCREASES ($V_R = IR$)
- V_L DECREASES (KVL)
- $\boxed{\frac{\Delta I}{\Delta t} \text{ DECREASES}}$

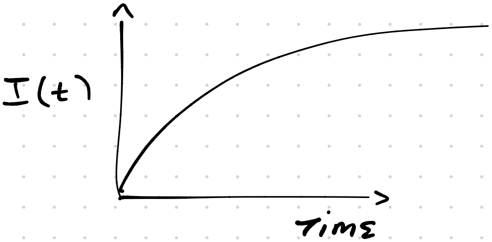
Ⓓ $\left(V_L = L\frac{\Delta I}{\Delta t}\right)$



- SO CURRENT IN R.L. SERIES CIRCUIT "CHARGES up" JUST LIKE CHARGE IN AN RC SERIES CIRCUIT:

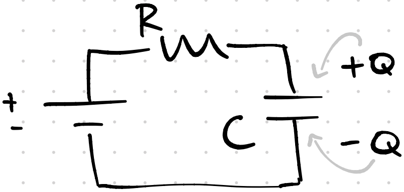
RL:

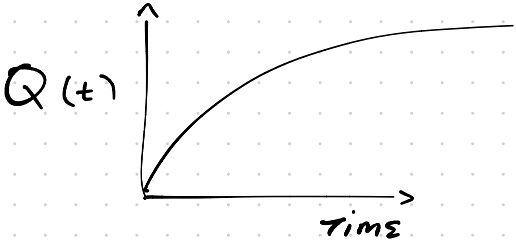




$$\tau = L/R$$

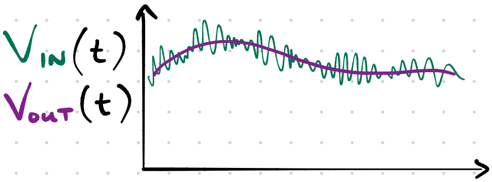
RC:





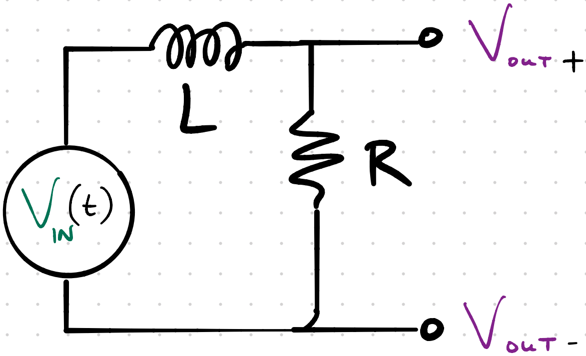
$$\tau = RC$$

- Application: Filtering Noise
  - Suppose we want to eliminate noise from an input signal before sending it to an output:



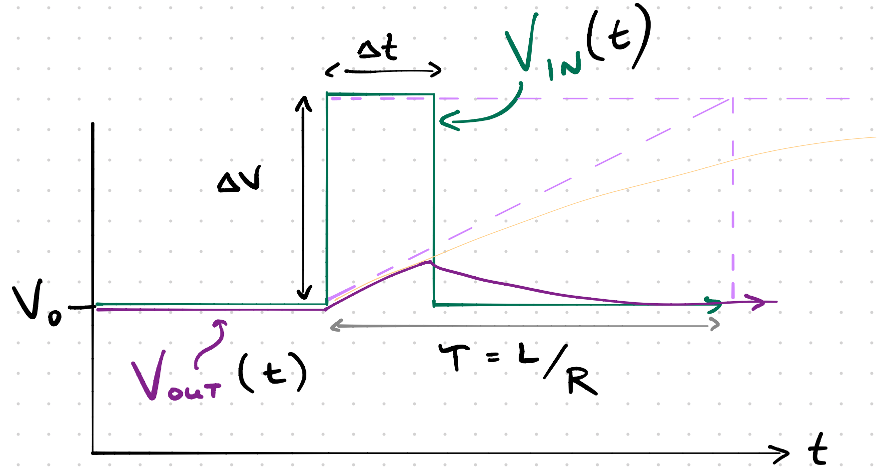
- From Input (Noisy)
- To Output (0 Noise)

- We can eliminate rapidly varying noise (∿∿∿), leaving only the smoothly varying signal (∼) using a "Low-pass" RL filter:

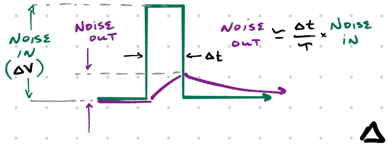


$$V_{OUT}(t) = V_{OUT+}(t) - V_{OUT-}(t) = V_R(t)$$

- To see how this works, let's consider a signal that is a steady voltage V_0 and "Noise" consisting of a bump of amplitude ΔV + duration Δt:



- Because T much longer than than Δt, current (and thus $V_R = IR$) does not get far along its "charging curve" before Noise Bump is over.



- Noise Bump reduced by factor $\simeq \frac{\Delta t}{T}$!