LECTURE 16 - KIRCHOFF'S RULES & RC CIRCUITS

SUMMARY

- KIRCHOFF'S VOLTAGE RULE: THE SUM OF THE VOLTAGE DROPS ACROSS ANY TWO PATHS W/ THE SAME START & END POINTS IS THE SAME.

- KIRCHOFF'S CURRENT RULE: THE CURRENT ENTERING A JUNCTION EQUALS THE CURRENT EXITING.

- CAPACITOR CURRENT: THE INCREASE IN CHARGE ΔQ DUE TO A CURRENT I CHARGING A CAPACITOR FOR A TIME Δt IS:

$$\Delta Q = I \Delta t$$

- A CAPACITOR W/ CAPACITANCE C CHARGES / DISCHARGES THROUGH A RESISTOR W/ RESISTANCE R W/ A CHARACTERISTIC "TIME CONSTANT" $\tau = RC$

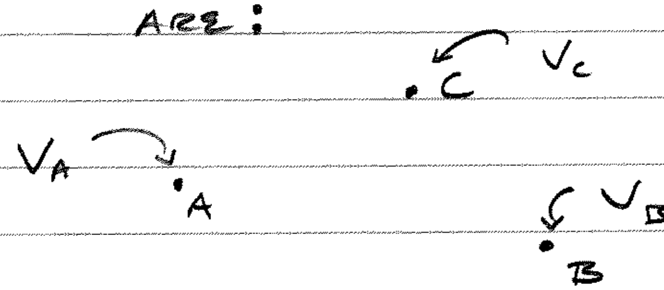
CHARGING: $V(t) = V\left(1 - e^{-t/\tau}\right)$

DISCHARGING: $V(t) = V e^{-t/\tau}$

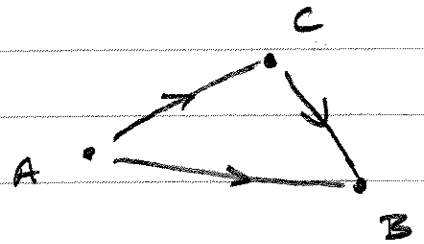
- IN "STEADY STATE" (ALL CURRENTS / VOLTAGES SETTLE TO STEADY VALUES), NO CURRENT ENTERS / EXITS A CAPACITOR AND IT LOOKS LIKE AN OPEN CIRCUIT.

16.1 KIRCHOFF'S CIRCUIT RULES

- ELECTRIC POTENTIAL IS A PROPERTY OF WHERE YOU ARE:



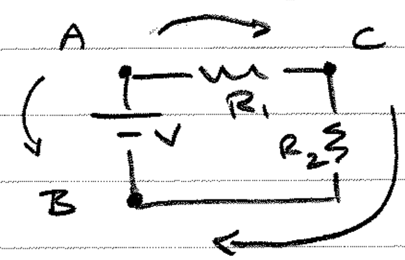
- THE DIFFERENCE IN ELECTRIC POTENTIAL BETWEEN TWO ENDS OF A PATH THEN DEPENDS ONLY ON THE END POINTS AND NOT ON THE POINTS IN BETWEEN:



$$\Delta V\,(A \to B) = V_A - V_B$$

$$\begin{aligned}\Delta V\,(A \to C \to B) &= \Delta V\,(A \to C) + \Delta V\,(C \to B)\\ &= \left[V_A - V_C\right] + \left[V_C - V_B\right]\\ &= V_A + \left[-V_C + V_C\right] - V_B\\ &= V_A - V_B = \Delta V\,(A \to B)\end{aligned}$$

- IN A <u>CIRCUIT</u>, E.G.



$$\begin{aligned}\Delta V\,(A \to B) &= \Delta V\,(A \to C \to B)\\ &= \Delta V\,(A \to C) + \Delta V\,(C \to B)\\ V &= V_1 + V_2\end{aligned}$$

"THE VOLTAGE DROP OVER R_1 PLUS THE VOLTAGE DROP OVER R_2 EQUALS THE VOLTAGE DROP OVER THE SOURCE V."

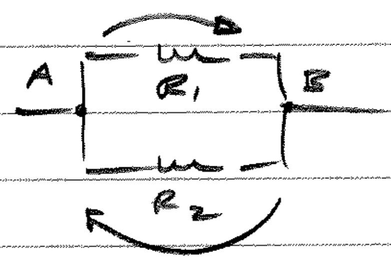
IN GENERAL:

KIRCHOFF'S VOLTAGE LAW (KVL)

"THE SUM OF THE VOLTAGE DROPS ACROSS ANY TWO PATHS W/ THE SAME START & END POINTS ARE EQUAL."

- WE HAVE IMPLICITLY USING KVL ALREADY:

PARALLEL RESISTORS:



$$\Delta V(A \rightarrow A) = \Delta V(A \rightarrow B \rightarrow A)$$

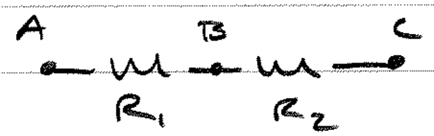
$$0 = V_1 + (-V_2)$$

(ARROW IS BACKWARDS!)

$$\rightarrow \quad V_2 = V_1$$

"VOLTAGE ACROSS PARALLEL RESISTORS ARE EQUAL"

SERIES RESISTORS:



$$\Delta V \,(A \rightarrow C) = \Delta V \,(A \rightarrow B \rightarrow C)$$

$$V = \Delta V\,(A \rightarrow B) + \Delta V\,(B \rightarrow C)$$

(VOLTAGE ACROSS SERIES) $= V_1 + V_2$

$\longrightarrow \quad V = V_1 + V_2$

"VOLTAGE ACROSS SERIES COMBINATION IS SUM OF VOLTAGE ACROSS EACH."

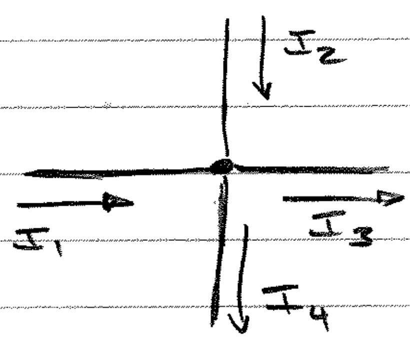
- PHYSICAL ORIGIN OF KVL:
  - $qV = U$ ← POT. ENERGY
  - BUT WE COULD ONLY DEFINE POTENTIAL ENERGY WHEN FORCES WERE CONSERVATIVE
    $\longrightarrow$ ENERGY CONSERVATION
- KVL IS A CONSEQUENCE OF <u>ENERGY</u> <u>CONSERVATION</u>!

KIRCHOFF'S CURRENT LAW (KCL):

- KVL IS CONSEQUENCE OF ENERGY CONSERVATION
- KCL IS " " CHARGE " !

KCL: "THE CURRENT ENTERING A JUNCTION EQUALS THE CURRENT EXITING THE JUNCTION".

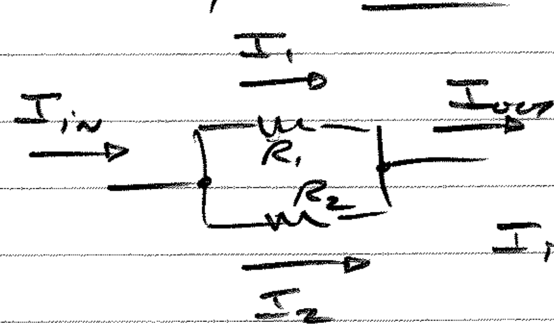
E.G.



$$I_1 + I_2 = I_3 + I_4$$

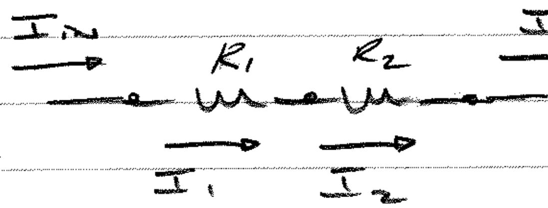
- AGAIN, WE HAVE BEEN USING THIS RULE IMPLICITLY ALREADY:

PARALLEL RESISTORS:



$$I_{IN} = I_1 + I_2 = I_{OUT}$$

SERIES RESISTORS



$I_{IN} = I_1 = I_2 = I_{OUT}$

- BUT! CHARGE CONSERVATION ONLY TELLS US THAT CHARGE CAN NOT BE CREATED OR DESTROYED.

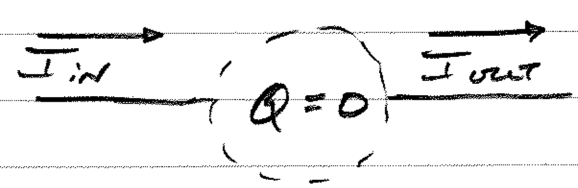
  - WHY DO WE REQUIRE

?

- CAN'T WE HAVE CHARGE <u>BUILDING</u> UP / ACCUMULATING IN SOME REGION?

<u>E.G.</u>

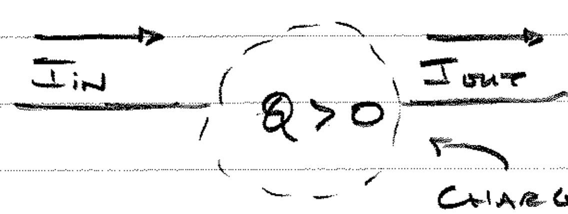
BEFORE:



$I_{IN} > I_{OUT}$

time $= t_1$

AFTER:



time $= t_2 > t_1$

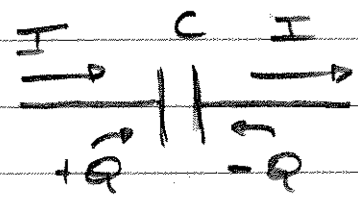
CHARGE BUILDS UP!

- ANSWER: YES, BUT WE ALREADY <u>HAVE</u> A CIRCUIT ELEMENT THAT STORES CHARGE! A <u>CAPACITOR</u>!

<u>16.2: RC CIRCUITS</u>

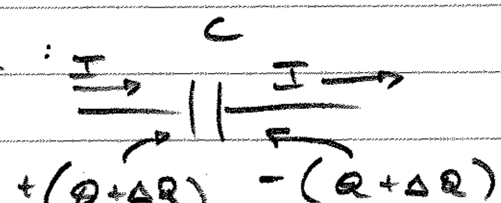
- LIKE WE CAN HAVE CURRENT THROUGH RESISTORS, WE CAN HAVE CURRENT CHARGING / DISCHARGING A CAPACITOR:

<u>BEFORE</u>:



time t

<u>AFTER</u>:



time $t + \Delta t$

BUT SINCE I IS $\frac{\Delta Q}{\Delta t}$:

$$\Delta Q = I \Delta t$$

QUESTION: TWO CAPACITORS C_1 + C_2, INITIALLY UNCHARGED, ARE CHARGED BY THE SAME CURRENT I FOR A TIME Δt. IF $C_2 > C_1$, WHICH CAPACITOR HAS THE HIGHER VOLTAGE?

ANSWER: $\Delta Q_1 = I \Delta t = \Delta Q_2$

BUT $V = Q/C$ FOR CAPACITORS SO:

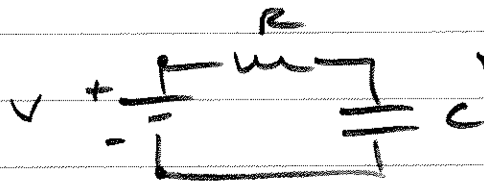
$$\Delta V_1 = \frac{\Delta Q_1}{C_1} = \frac{I \Delta t}{C_1} > \frac{I \Delta t}{C_2} = \frac{\Delta Q_2}{C_2} = \Delta V_2$$

$\longrightarrow \Delta V_1 > \Delta V_2$

- CHARGING A CAPACITOR THROUGH A RESISTOR:

  - SUPPOSE @ A TIME $t=0$ WE HAVE THE FOLLOWING CIRCUIT:

@ $t=0$, $Q=0$ (UNCHARGED)



- WHAT HAPPENS FOR $t > 0$?

- BY KVL:

$$V = V_R + V_C$$

@ $t=0$, THOUGH, $Q=0$ SO $V_C = 0$

$\longrightarrow V_R = V$ (@ $t=0$)

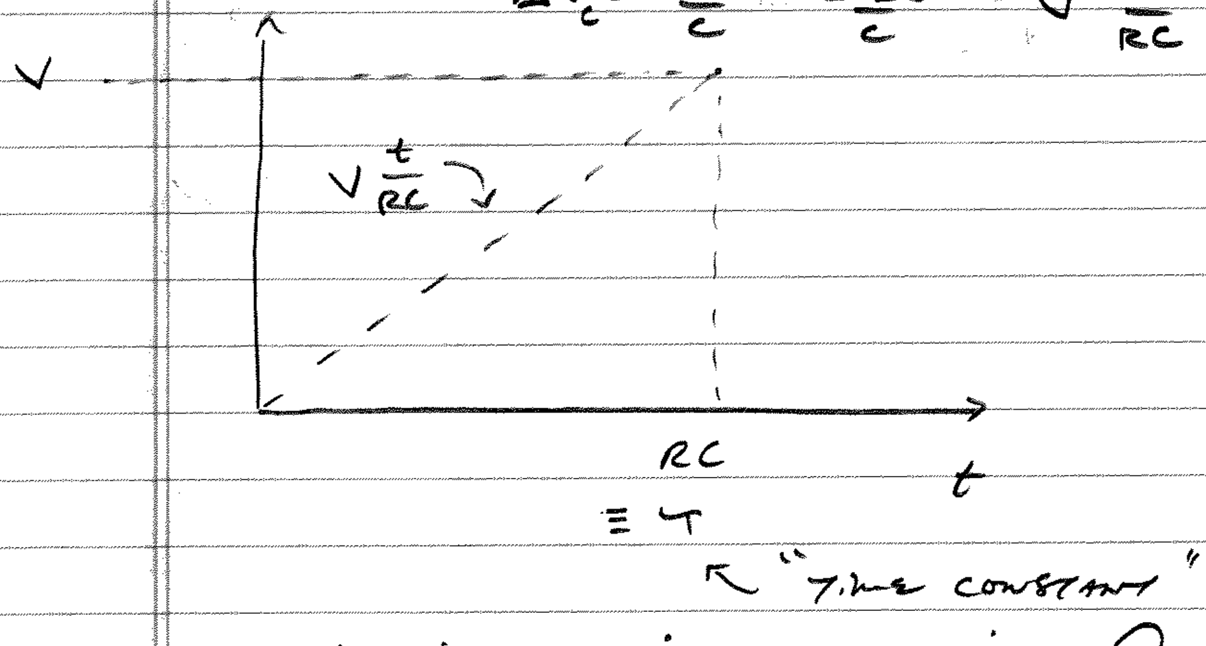
- BY OHM'S LAW:

$$I = V_R/R$$
$$= V/R \quad [@\ t=0]$$

- BY KCL:
  - CURRENT THROUGH RESISTOR IS SAME CAPACITOR (THEY ARE IN SERIES).

- THEREFORE @ $t=0$ CAPACITOR BEGINS CHARGING @ RATE $I = V/R$

$$\Delta V_c = \frac{\Delta Q}{C} = \frac{I \Delta t}{C} = V \frac{\Delta t}{RC}$$

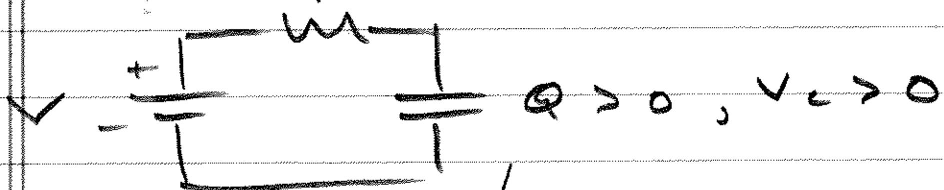


- IF IT CONTINUES CHARGING @ THIS RATE, WILL REACH BATTERY VOLTAGE V IN A TIME $\tau = RC$, CALLED THE "TIME CONSTANT" OF THE CAPACITOR/RESISTOR COMBO.

- AS THE CAPACITOR CHARGES, THE VOLTAGE ACROSS IT INCREASES, WHICH, BY KVL, DECREASES THE VOLTAGE ACROSS THE RESISTOR

    - THIS DECREASES THE CURRENT THROUGH THE RESISTOR, WHICH, BY KCL, DECREASES THE RATE @ WHICH WE CHARGE THE CAPACITOR:

$t > 0$:



$$I = V_R/R < V/R = \text{current @ } t=0$$

- AFTER A LONG TIME, WHEN WE REACH STEADY STATE (ALL VOLTAGES STEADY), THE CURRENT GOES TO ZERO (OTHERWISE V_C WOULD BE CHANGING IN TIME).

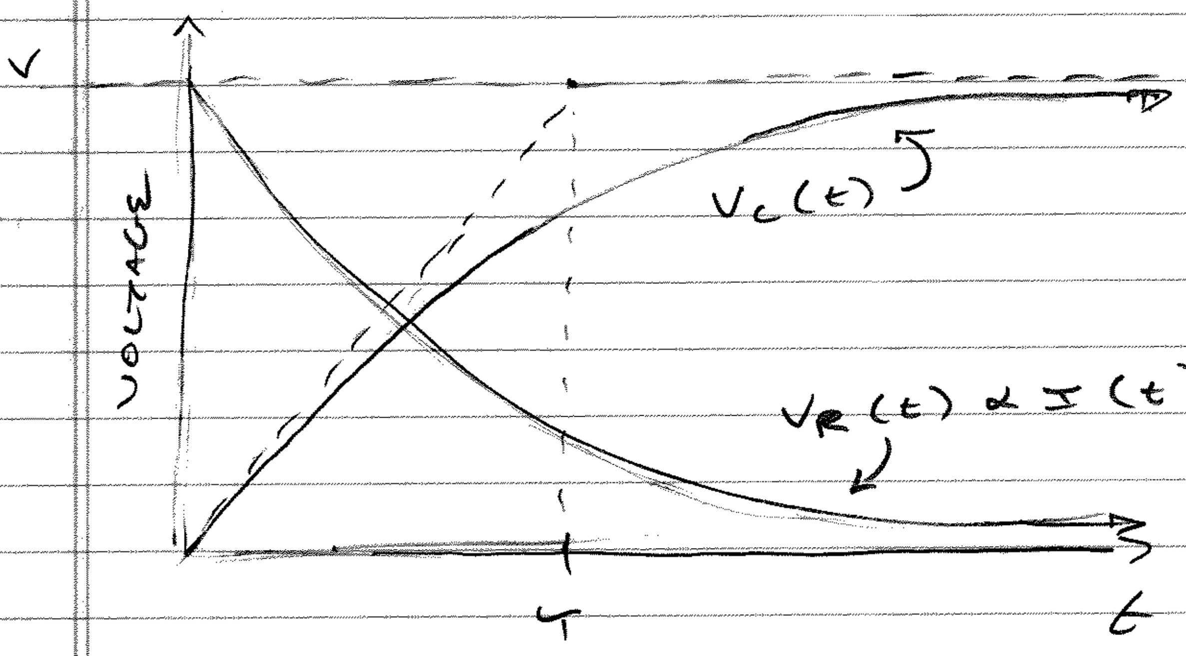
    - IF $I = 0 \rightarrow V_R = 0$ (OHM'S LAW)
    - IF $V_R = 0 \rightarrow V_C = V$ (KVL)

- NOTE: SINCE IN STEADY STATE THE CURRENT INTO A CAP MUST BE ZERO, THEN IN S.S. A CAP ACTS LIKE AN OPEN CIRCUIT.

- PUTTING THIS TOGETHER:
  - @ $t=0$ $(Q=0)$, CAP BEGINS CHARGING @ RATE

$$\frac{\Delta V}{\Delta t} = \frac{V}{\tau}$$

  - AS $t>0$, RATE OF CHARGING GRADUALLY DECREASES.
  - EVENTUALLY, CAPACITOR VOLTAGE SETTLES TO VOLTAGE V OF SOURCE:



- MATHEMATICALLY:

$$V_c(t) = V\left(1 - e^{-t/\tau}\right)$$

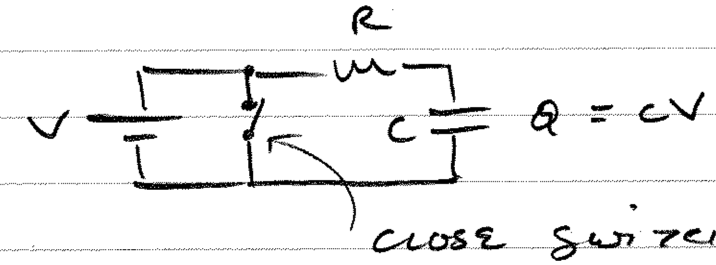
"EXPONENTIAL CHARGING"

$$V_R(t) = V e^{-t/\tau}$$

"EXPONENTIAL DECAY"

- QUESTIONS FOR YOU:

AFTER THE CAPACITOR CHARGES UP TO V, WE "SHORT OUT" THE VOLTAGE SOURCE:



- WHAT HAPPENS TO V_C AFTER WE CLOSE THE SWITCH?
  - V_R?