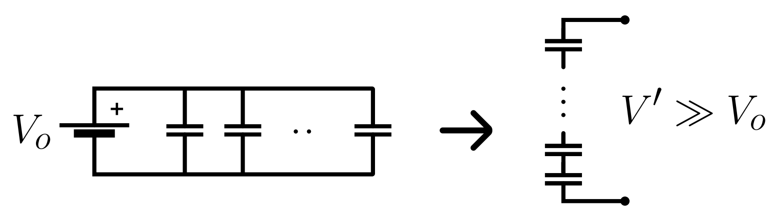
302L s20 leture 13 - marx generators and sting rays

1 Introduction

This lecture serves as a conclusions to our study of electrostatics (Serway Chapters 15 and 16). In this lecture we will look at two applications where a high voltage is obtained by charging a large number of capacitors at a low voltage and then rearranging their connections so that they are connecting in series:



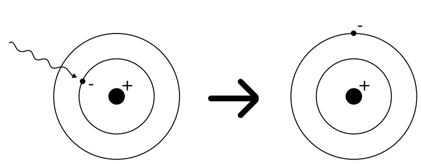
One of these applications is used in science and technology and is referred to as a Marx bank (or Marx generator). The other application occurs in the electric organ of electric fish (electric eels, sting rays).

2 Two photon absorption, Pockels cells, and Marx banks

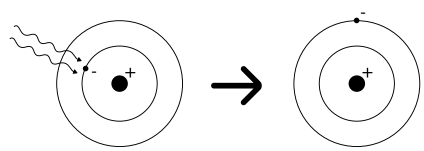
Note: the discussion below about two-photon absorption and the Pockels cell is just for fun. It is included to show you why a Marx bank would be useful. The Marx bank itself is the topic that applies concepts you have learned so far in class.

2.1 Two photon absorption

When we typically think of individual atoms absorbing light on the microscopic scale we think of the atom absorbing a single photon. The energy in the photon is used to move an electron in the atom to an excited orbital:

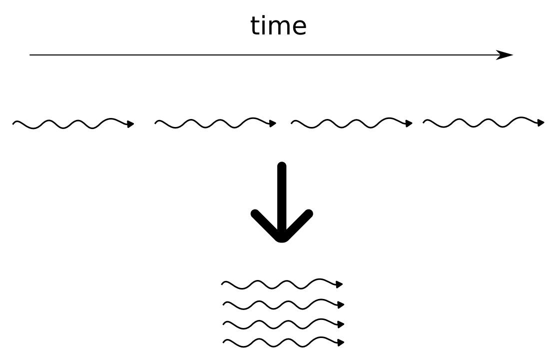


However, there is nothing forbidding the possibility of two photons getting *simultaneously absorbed* and accomplishing the same things:

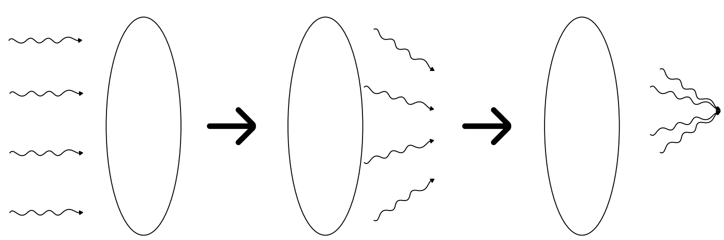


In everyday circumstances we can ignore this possibility. At typical light intensities the likelihood of two photons being in close enough proximity to accomplish this is simply too low.

Scientists that wish to study two-photon absorption therefore need some way to cram a lot of photons into a small region of space. In other words they need a beam of light with a very high *intensity*. One way of getting this is to take the energy in a typical beam of light and squeeze it into a very short time interval:



If we take, say, 1 Joule of energy, which is the amount of energy your hand absorbs by pointing it at the sun for 1 second, and compress it all into $10\,\mathrm{ns} = 10 \times 10^{-9}\,\mathrm{s}$, then we obtain $P = \frac{E}{\Delta t} = 100\,\mathrm{MW}$ in power. We can further increase the proximity of the photons in our pulse if we also *focus* the light with a lens:

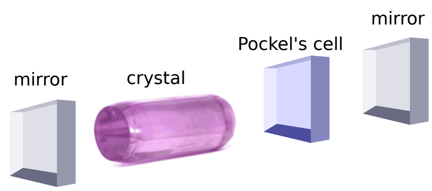


Then we can obtain the kinds of intensities necessary to observe two-photon absorption.

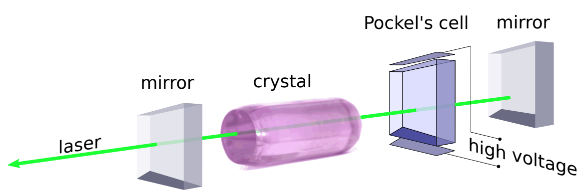
2.2 Pockels cell

To squeeze light into a very short time duration we use the following trick. We first store some energy in the atoms of a crystal:

This is accomplished by illuminating the crystal with a flash lamp (see lecture 11 - capacitor energy). Photons from the flashlamp get absorbed by atoms in the crystal, leaving the atoms in an excited state. Compared to the intensities we're trying to achieve, the flashlamps are relatively weak. To put enough energy in the crystal we need to turn the flashlamps on for about a millisecond, which is 10^5 times longer than our laser pulse duration. To prevent the energy from leaving the crystal during this time, a special optic called a "Pockels cell" is placed near the crystal inside of a laser cavity (two mirrors):



If a high voltage is applied between two ends of the Pockel's cell:



a large electric field is then generated in the Pockel's cell which modifies its optical properties, and allows the energy stored in the crystal to be rapidly ($\approx 10\,\text{ns}$) converted into laser light.

2.3 Marx bank

The voltage we need to apply across the Pockels cell is about 5 kV. There are certainly power supplies capable of generating this potential[1], but they do not work for this application because we can't turn them on fast enough. When we flick the power supply on, the voltage supply will build up slowly from 0V to its full potential, and the energy in the crystal would slowly leak out and we would not get the high intensities necessary to study two-photon absorption.

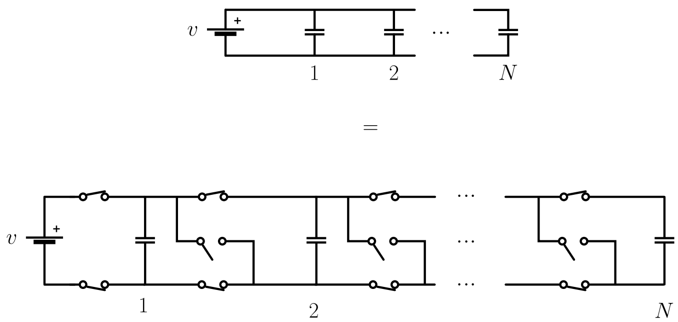
We need a way switch the voltage across the cell from 0V to 5kV in 10ns. This is typically accomplished by using a *Marx bank*, named after its inventor, the German electrical engineer Erwin Marx.

The Marx bank works by using a small voltage v to charge up a large number (N) of capacitors in parallel and then rapidly switching the connections between the capacitors so that they are now all in series. The result is that the small voltages v across each of the N capacitors add together to produce a large

[1] The neon signs down at Posse East all use voltage sources of about 10 kV.

voltage $V' = Nv \gg v$ across their series combination (see the diagram at the very beginning of the lecture).

When during the charging stage, the Marx bank looks like this:

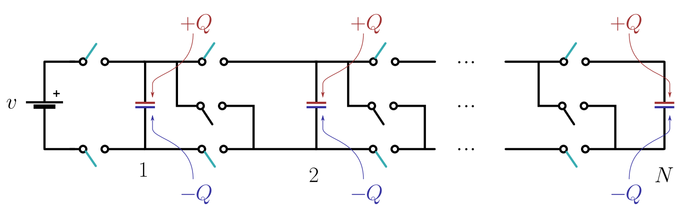


The actual circuit is the lower diagram. If we take note of which switches are open and which are closed, we find it is electrically equivalent to the upper diagram[2]. Since the capacitors are connected in parallel, the voltage across all of them is the same (why?). By inspecting the diagram we see that they are connected to the positive and negative end of the voltage source, so that the voltage across the capacitors is v. Therefore the charge $\pm Q$ on each capacitor is

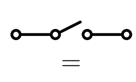
$$Q = Cv$$

where C is the capacitance of each individual capacitor.

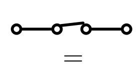
Once the capacitors are charged up and we are ready to generate a high voltage, we open all the switches that were formerly closed:



[2]If this part is not clear, here is a more detailed explanation. Recall that an *open* switch is electrically equivalent to a capacitor with a very small capacitance c. We can therefore eliminate it from the diagram, since anything else connected in parallel with it will have a much larger capacitance $C \gg c$ and thus the open swich contributes negligibly to their combined parallel capacitance ($C' = C + c \approx C$). Schematically:

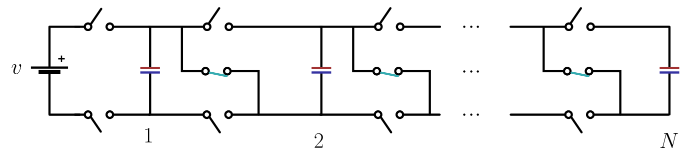


Similarly, a *closed* switch joins two conductors with another conductor, forming a single continuous conductor. Therefore:

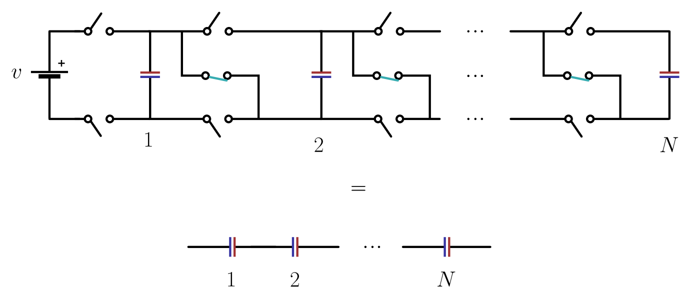


To aid the eye we've colored all the switches that were opened in teal. Looking at the diagram now we see that all each capacitor is now *isolated* in that there is no conductive connection between any two capacitors. Each capacitor still holds the charge $\pm Q = \pm Cv$ that accumulated during the charging stage.

Now we will close the switches connecting the top ends of a capacitors to the bottom ends of the capacitors to their right:



The switches that are now closed are colored in teal. Simplifying the circuit as we did before (open switches eliminated, closed switches replaced with wires), we obtain:



We are now finished rearranging our connections and we find we have taken our N capacitors that were originally connected in parallel and connected them in series. To determine the voltage V that across this series connection we make a couple observations:

- The left conductor of the first capacitor contains a charge $-Q$.
- The right conductor of the last capacitor contains a charge $+Q$.
- Each of intermediate conductors contains a charge $+Q$ and $-Q$ on their left and right ends, respectively, so that overall they contain no net charge.

In light of these observations, we can conclude that we end up with a series capacitor of capacitance

$$C' = \left(\frac{1}{C} + \frac{1}{C} + \cdots + \frac{1}{C}\right)^{-1} = \left(\frac{N}{C}\right)^{-1} = \frac{C}{N}$$

that contains a charge $Q = Cv$ equal to the original charge stored on each of the capacitors[3]. Therefore the voltage V across the series combination is

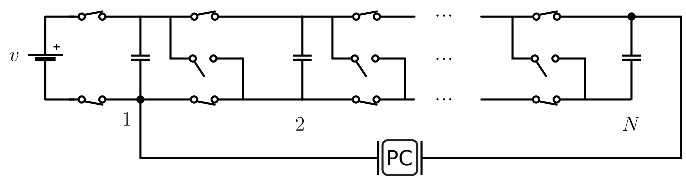
$$V = \frac{Q}{C'} = \frac{Cv}{C/N} = Nv$$

so we find that indeed we get the factor N multiplication of our original charging voltage v across our series combination.

[3] Note that the last bullet point in the list above is important in order to reach this conclusion. If any of the intermediate conductors contained charge then the series combination would not behave as a capacitor with capacitance C'.

2.4 Marx bank epilogue

To tie the discussion back to the application of the Pockels cell and two-photon absorption, let's suppose we have our Marx bank connected in the parallel configuration and a Pockels cell connected to the negative end of the first capacitor and the positive end of the last capacitor:

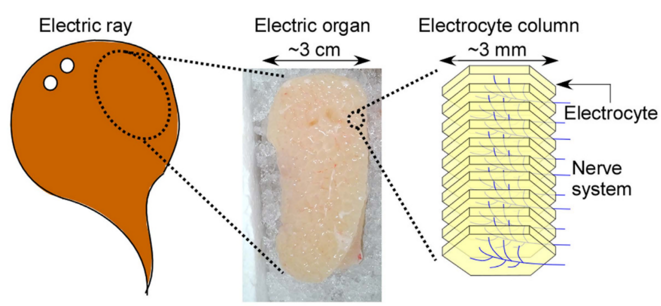


In this configuration, we can allow as much time as we need (seconds, minutes, whatever) for the voltage supply to charge up the capacitors. During the charging period, the voltage across the Pockels cell changes from zero to v. If N is large, then v is much smaller than the voltage $V = Nv$ necessary to activate the Pockels cell. Therefore the Pockels cell is mostly essentially unaffected during the charging interval.

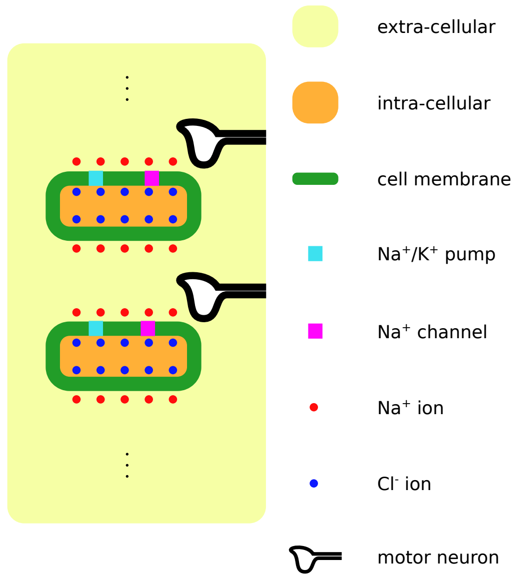
To activate the Pockels cell we must configure the switches in the Marx bank so that the capacitors are connected in series. The speed with which we activate the Pockels cell is then determined by how rapidly we can toggle the switches. Using electronic switches this toggling can be achieved in an extremely short period time, i.e. roughly 10ns, which is roughly the duration of the laser pulse we are trying to generate.

3 Electric fish

The Marx bank, interestingly enough, was in some sense invented much earlier by the mechanism of natural selection. Electric fish (electric eels, electric rays, etc.) contain special organs called electric organs. The organs contain stacks of cells called electrocytes (or electroplaques). The electrocytes in a given stack work collectively to generate a large difference in electric potential between the two ends of the stack. See the figure below for an illustration:



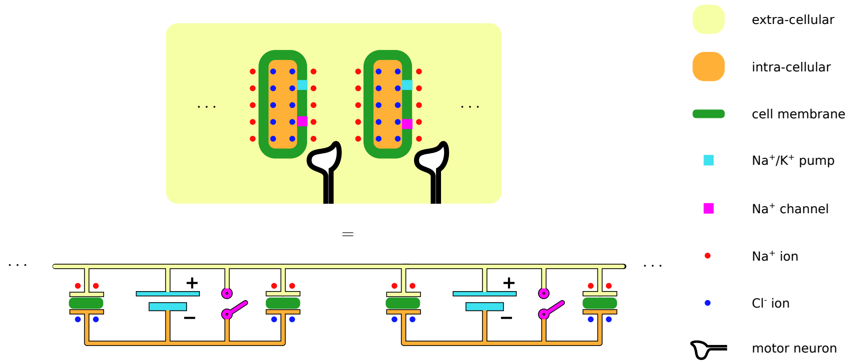
Let's zoom in so that we can see two of these electrocytes along some stack:



This is certainly a simplified picture of the electric organ but it suffices for the purpose of explaining its electrical properties. Breaking down each component:

- **extra-cellular fluid**: A conductive fluid filling the space between cells.
- **intra-cellular fluid**: A conductive fluid filling the space inside the cells.
- **cell membrane**: An insulating material separating the inside and outside of the cells.
- **Na^+/K^+ pump**: A protein that consumes ATP and transports positive charge from inside the cell to the outside.
- **Na^+ channel**: A protein that can be activated by a neurotransmitter (acetylcholine) to transition from a closed (non-conducting) state to an open (conducting) state. Na^+ ions transported out of the cell by the Na^+/K^+ pump can reenter the cell via the Na^+ channel when it is in its open state. It is crucial to note that only side of these cells possess these Na^+ channels (i.e. the top sides in the above illustration).
- **Na^+/Cl^- ions**: In normal cellular operation, the Na^+ channels are closed. The action of the Na^+/K^+ pump produces an accumulation of positive Na^+ ions on the outside of the cell, leaving behind an excess of negative Cl^- ions on the inside.
- **motor neuron**: A motor neuron is in close proximity to the side of the cell containing the Na^+ channels. The motor neurons release acetylcholine to trigger the opening of the Na^+ channels.

From this picture of the electric organ we form the following electrical model:

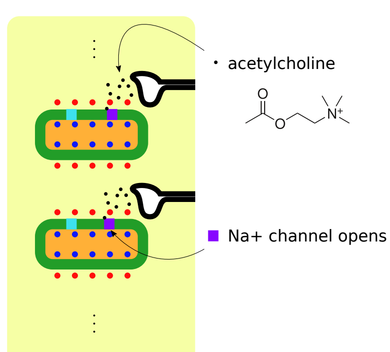


Here are some notes explaining the correspondance between the cell diagram and its electrical equivalent:

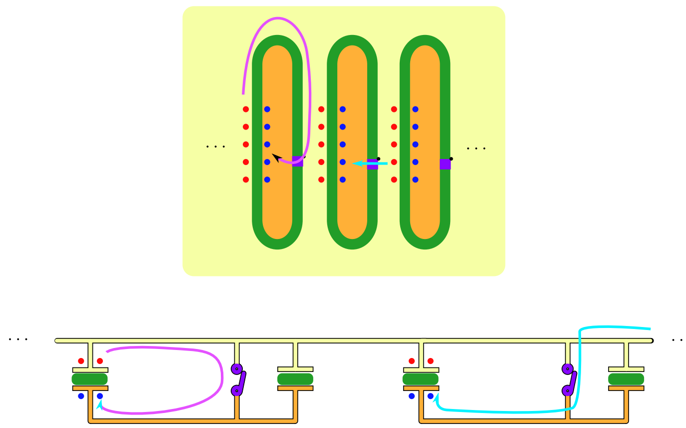
- We treat each side of the cell as a capacitor with one conductor the intra-cellular fluid and the other being the extra-cellular fluid just outside the cell. The cell membrane acts as the insulating dielectric filling the space between these two conductors.
- The Na^+/K^+ pump acts as a voltage source or battery which acts to keep the intra-cellular fluid at a lower electric potential than the extra-cellular fluid.
- When the Na^+ channels are unactivated (closed), the do not permit the flow of charge between the inside and outside of the cell, and thus act as *open* switches. Note the potentially confusing relationship here (closed channel = open switch).
- The extra-cellular fluid between the cells acts as a conductive connection keeping the fluid outside of all the cells at the same potential.

In particular, the last bullet point item implies that difference in electric potential between any two points in the electric organ can be no larger than the potential difference between the inside and outside of a single cell. This "membrane potential" for electrocytes is about 150mV, which is large compared to the membrane potential of other cells but still puny compared to the voltages necessary to shock or stun predators or prey.

To generate a large potential, the fish's brain initiates the release of acetylcholine from the motor neurons, which opens the Na^+ channels (shown in purple now to indicate change in state):



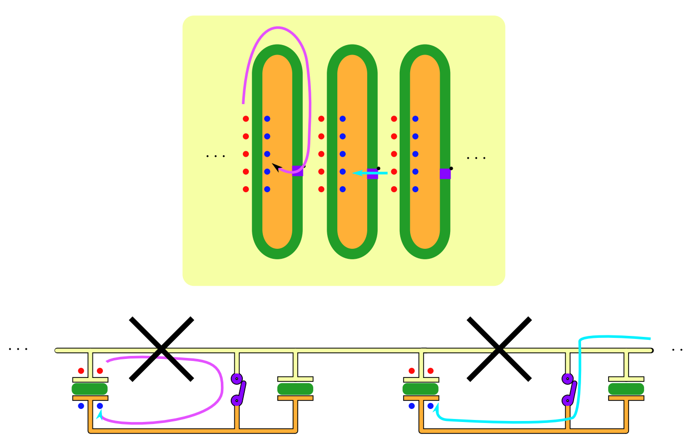
The opening of the Na^+ channels is equivalent to the closing of our electrical switches in our corresponding circuit:



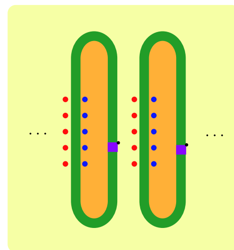
In addition to showing the switches as now closed, we have made a couple other modifications to the diagram:

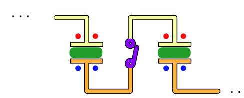
- The rearrangement of charge triggered by the opening of the Na^+ channels is much more rapid than the Na^+/K^+ pumps can keep up with. They are effectively useless over these short time scales and so we have eliminated them to simplify the picture.
- The Na^+ ions on the sides of the cell containing the Na^+ channels will instantly travel through the channels to the inside of the cell where the electric potential is lower. We therefore remove the red and blue dots from the capacitor representing the side of the cell containing the Na^+ channels.

The opening of the Na^+ channels, also provides a path (shown in pink) for the Na^+ ions on the *other* side of the cell to enter. This path, however, is much longer than the alternative path (shown in light blue), where Na^+ ions move to the inside of a *neighboring* cell. It is therefore not a bad approximate to treat the region of the extracellular fluid along the former (pink) path as an *open circuit*:



Making this modification, we obtain the following equivalent circuit for the electric organ after the release of acetylcholine:





In the updated schematic we have eliminated the discharged capacitor[4]. What we end up with is electrically equivalent to the final configuration of the Marx bank; that is, we obtain a series connection of capacitors with opposite charges. The potential difference between the two ends of a stack of electrocytes is therefore the *sum* of the membrane potentials of all the electrocytes in the stack.

In a “strongly electric” fish the number of electrocytes in a single stack can be quite large. For example, the electric eel possesses stacks containing as many as 6,000 electrocytes. These stacks amplify the small 150mV membrane potential of each electrocyte into a powerful shock of roughly 1kV! What would be the effect/purpose of connecting multiple such stacks in *parallel*?

[4]We can make this simplification because the discharged capacitor is not really a capacitor anymore. Its two conductors (the intra- and extra-cellular fluids) are joined together by another conductor (the Na^+ channels), so that is no longer two isolated conductors but one single conductor.