302L S20 Lecture 12 - Dielectrics

1 Summary

- Filling the space between the plates of a parallel plate capacitor with an insulating material increases the capacitor's capacitance by a factor ϵ_r known as the insulator's *relative permittivity*. An insulator that is used for this purpose is known as a *dielectric*.
- The magnitude of a (non-polar) dielectric's relative permittivity is determined by the *polarizability* α of the atoms composing the dielectric. A dielectric containing a high density of highly polarizable atoms will have a large relative permittivity. Specifically we have the relation

  $$\epsilon_r = 1 + \frac{n\alpha}{\epsilon_o}$$

  where n is the number of atoms in the dielectric per unit volume[1].
- The polarizability α of an atom is given by the ratio $\frac{p}{E}$ where E is the electric field at the atom's location and p is the dipole moment induced by the molecule by the electric field.
- The dipole moment p of an atom and measures the degree of charge separation induced in the atom by some electric field. It is equal to the product δq, where q is the charge of the atom's nucleus and δ is the displacement of the atom's negatively charged electron cloud from its positively charge nucleus.
- Conceptually we can think of a dielectric as "screening" the charge on the capacitor plates it lies between, reducing the charge by a factor $1/\epsilon_r$.

2 Introduction

In the first lecture of the semester we introduced conductors and insulators as a way of categorizing materials in terms of the mobility of their charge carriers. Conductors you will recall are the materials where the charge carriers move freely, while insulators contain charge carriers that are strongly bound. In insulators charge carriers are only able to displace themselves microscopically small distances in response to electric fields.

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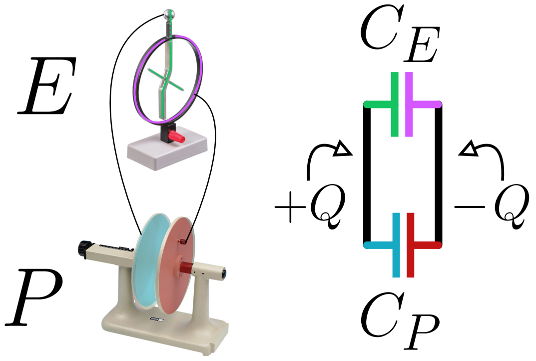
[1] Technically this relation only applies for relatively *dilute* materials but this complication does not concern us.

Since the first lecture conductors have received considerable attention. We learned for instance that conductors in equilibrium have no internal electric field (and therefore a constant electric potential), and that pairs of conductors can be used a capacitor to store charge. In this lecture we let the insulators regain some of the limelight as we introduce the topic of dielectrics.

3 Dielectrics

Consider again the arrangement of a parallel plate capacitor connected in parallel with an electroscope:



We've used the Wimhurst machine to place some positive and negative charge $\pm Q$ on each side of the combined parallel capacitor. The electroscope and the parallel plate capacitor each respectively take a portion $\pm Q_E$ and $\pm Q_P$ of this charge $\pm Q$ in proportion to their capacitance, i.e.

$$\frac{Q_E}{Q_P} = \frac{C_E}{C_P} \tag{1}$$

Recall that the resting position of the electroscope needle is vertical when the electroscope is uncharged. When the electroscope acquires charge, the needle deflects horizontally to increase its separation from the vertical bar since the two contain like charges[2]. The horizontal deflection of the needle is thus a measure of the charge Q_E on the electroscope.

Now we insert a piece of plastic, which is an insulator, between the plates of the electroscope. For illustration we've colored the plastic yellow in the diagram below:

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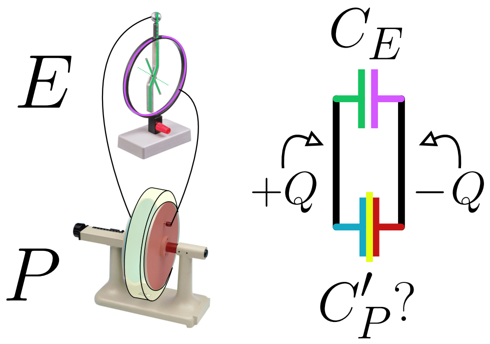
[2] Recall that the needle and the vertical bar are in conductive contact and together comprise one conductor of the capacitor E.



We find after inserting the plastic that the electroscope needle has reduced its horizontal deflection, indicating a loss in charge of the electroscope. Note that we can remove the plastic and the electroscope returns to its original horizontal deflection. This means that inserting the plastic did not remove charge from the system but only caused a transfer of charge off the electroscope and onto the parallel plate capacitor. This makes sense since after all the plastic is an insulator and is not be able to conduct electricity between the two plates of the capacitor.

Instapoll question:

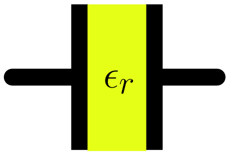
What does this result imply about how the capacitance C'_P of the parallel plate capacitor with the plastic compares to the capacitance C_P without the plastic?

Answer:

In light of equation (1), a decrease in the ratio $\frac{Q_E}{Q_P}$ implies a decrease in the ratio $\frac{C_E}{C_P}$ of the two capacitances. Since we did not modify the electroscope, C_E could not have changed, so we can conclude that C_P must have *increased*, i.e. $C'_P > C_P$. When an insulator is used for this purpose of increasing a capacitor's capacitance, it is referred to as a dielectric.

If we were to repeat the experiment using capacitors with different the plate areas A or plate separations d, we would find that filling the space between the two plates with a given insulator increases the capacitance by the same factor, which we denote by the symbol ϵ_r. In other words we have:

$$C = \frac{\epsilon_r \epsilon_o A}{d} \tag{2}$$



$$C = \frac{\epsilon_r \epsilon_o A}{d}$$

Further, we would expect this factor ϵ_r to depend on which insulating material we use, since the empty space originally filling the space between the plates is also certainly insulating. So we find then we can associate with every insulating material its correponding factor ϵ_r which we call its *relative permittivity* or, alternatively, its *dielectric constant*[3]. Here is a table of the relative permittivities of some different materials:

Material	C'/C
air (1 atm)	1.0006
air (100 atm)	1.055
vacuum	1.0
teflon	2.1
alumina	10.0
liquid argon (Ar)	1.54
water (room temperature)	80

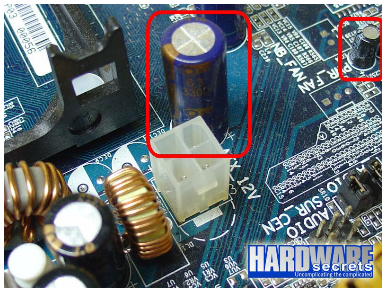
A couple observations:

- Some insulators can significantly modify capacitance. Alumina, for instance, can increase the capcitor's capacitance by a factor of 10.
- By looking at the relative permittivity of air at 1 atm and 100 atm, we find that the *increase* in capacitance seems to scale with the *density* of the insulator.
- Water, which is a liquid composed of *polar* molecules (H_2O), has a much higher dielectric constant than liquid argon, which is composed of *non-polar* molecules (Ar).

For the remainder of the lecture we will show an application of the first observation and give an explanation of the second observation. In the homework you will explore the third observation.

4 Aluminum Electrolytic Capacitor

As a practical application of dielectrics we can look at the aluminum electrolytic capacitor. You might recognize these guys if you've ever looked inside of a PC:



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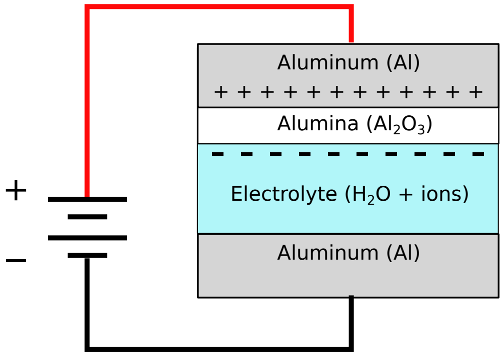
[3] Sometimes the symbol κ is also used in place of ϵ_r.

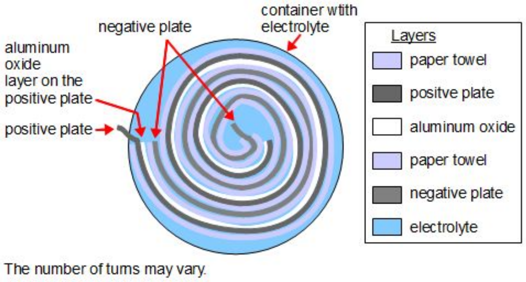
Electrolytic capacitors are a cost- and space-effective way to store lots of charge. Here is a very simplified diagram of what one looks like on the inside:



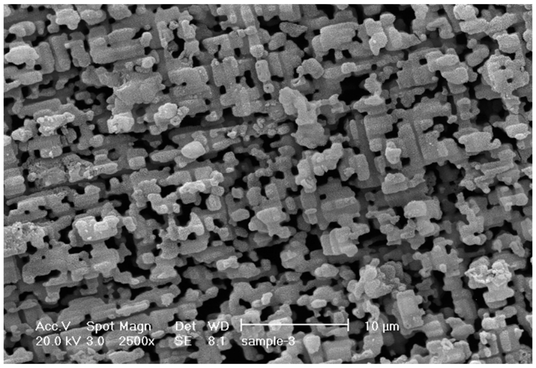
A conducting electrolyte (think salty water) fills the space between two conducting aluminum plates (foils actually). The surface of one of the aluminum plates has been *oxidized*, i.e. reacted with oxygen to produce a layer of aluminum oxide, also known as alumina. While pure aluminum is a pretty good conductor, alumina is an excellent insulator with a high relative permittivity.

Note that it is not the spacing between the aluminum foils which determines the plate spacing d, but rather it is the thickness of the oxide layer. This is because the electrolytic solution is, as we mentioned, conducting. Because the alumina dielectric layer can be made very thin (a few nanometers), we find that, in light of the equation (2), these capacitors can be made to have a very large capacitance.

For large capacitances we also want a large plate area A. Manufacturers accomplish this by cutting the pairs of foil into long rectangles and rolling them up and stuffing them into cylindrical cannisters:



To further increase the effective plate area A the foils are microscopically "roughened" using electrochemistry. Here is an electron microscope image of one of these roughened foils:



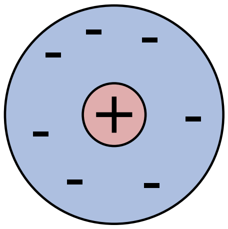
The pits and protrusions making up this rather strange "Martian landscape" on the foil's surface serve to increase the effective plate area and thus increase its ability to store charge. In some cases the capacitance using the roughened foil can be as much 200 times greater than the capacitance of a microscopically flat foil!

As an aside: from the scale bar on the image we see the size of the region shown is very small – only about $50\,\mu\text{m} \times 25\,\mu\text{m}$. This may be small compared to everyday objects, but keep in mind that the thickness of the alumina dielectric is 1000× smaller still!

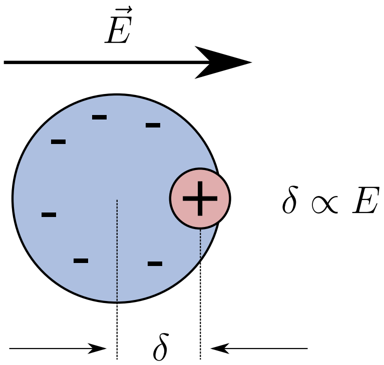
5 Microscopic origin of the relative permittivity ϵ_r

5.1 Atomic polarizability

To understand *how* dielectrics are able to increase capacitance, we must look at the material on a microscopic scale. The material is ultimately composed of atoms, which can be thought of as a postively charged nucleus surrounded by a cloud of negative charge formed by the atom's electrons:



We show here the arrangement of charge in the atom in the absence of an electric field. In this case the atom's most stable arrange is with the negative charge cloud centered perfectly around the positive nucleus. However, when we apply an electric field:



then the postively charged nucleus becomes pulled in the direction of the field, while the electron cloud gets pulled in the opposite direction, with the result that the two become displaced relative to one another by some distance δ. To increase this separation δ we must increase the electric field strength E, and to a very good approximation we can assume that the two are proportional to one another:

$$\delta \sim E$$

The proportionality factor relating the two is different for different atoms. For example, if a helium atom and a lithium atom are placed in the same electric field, then the displacement δ_{Li} for the lithium atom is about a hundred times larger than the displacement δ_{He} of the helium atom! Is this fact consistent with what you know about the *chemical* behavior of the two elements?

Instead of defining some coefficient β for each atom so that $\delta = \beta E$, we will soon see that it is more convenient to define for each atom a coefficient α known as the atom's *polarizibility* so that

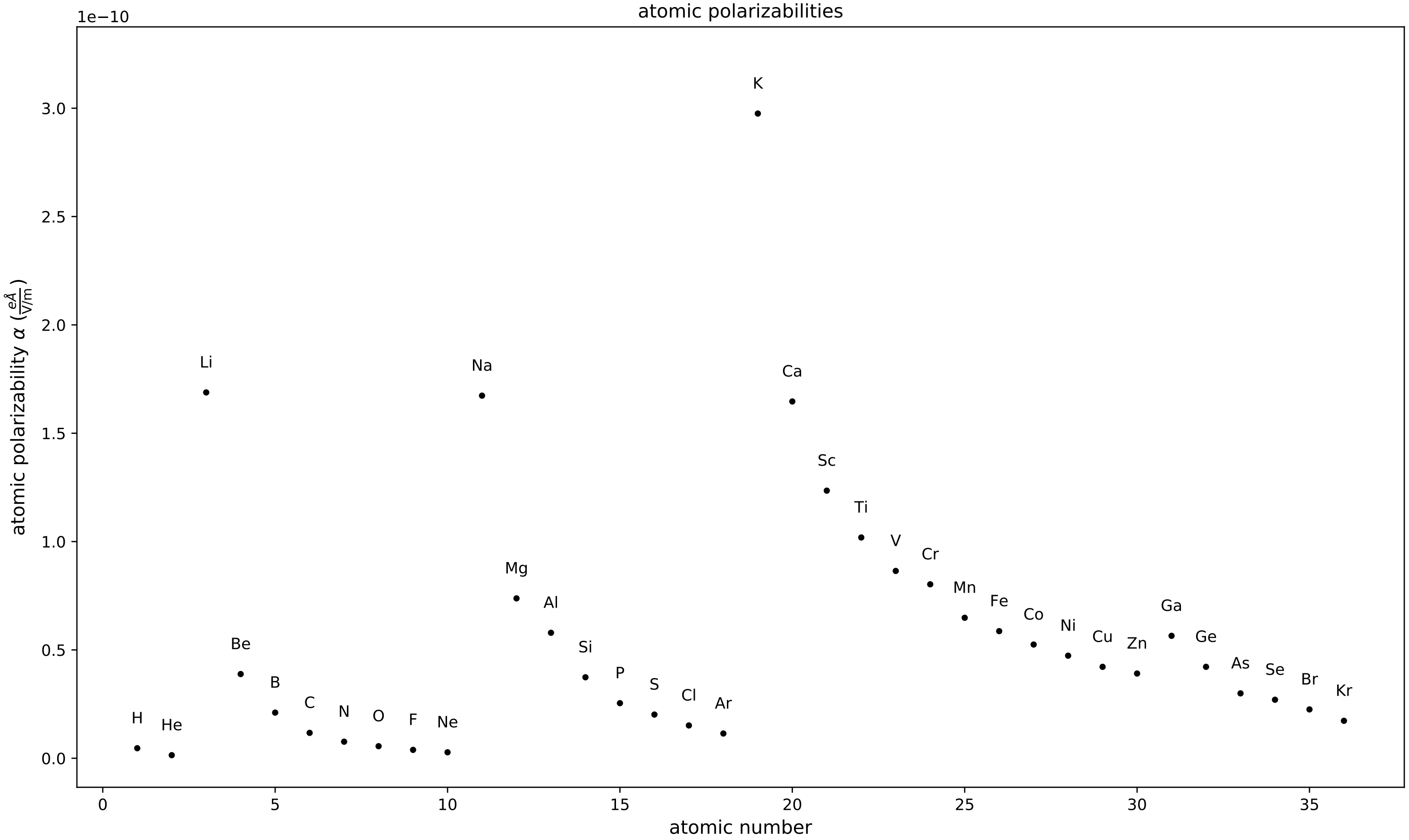
$$q\delta = \alpha E \tag{3}$$

where q is the charge of the nucleus (+1 for hydrogen, +2 for helium, etc.). The quantity $q\delta$ is known as the *dipole moment* of the atom and is denoted by the symbol p, i.e.

$$p \equiv q\delta \tag{4}$$

The dipole moment as we've just defined it appears to be a measure of the degree of *charge separation* or "polarization" of the atom. An atom with a large polarizability α is therefore more "polarizable" in the sense that a small electric field will induce in the atom a large polarization.

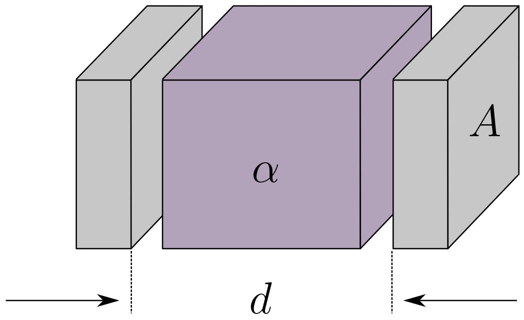
In figure **??** you can find a plot of the polarizabilities of the first 36 elements. What do you make of the "periodic" rapid increases followed by slow decreases?


atomic polarizabilities
1e−10
atomic polarizability α ($\frac{e\text{Å}}{\text{V/m}}$)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30
35
atomic number
H
He
Li
Be
B
C
N
O
F
Ne
Na
Mg
Al
Si
P
S
Cl
Ar
K
Ca
Sc
Ti
V
Cr
Mn
Fe
Co
Ni
Cu
Zn
Ga
Ge
As
Se
Br
Kr

5.2 Polarizability and permittivity

What is the connection between a dielectric's relative permittivity ϵ_r and the polarizability α of the atoms making up the dielectric? To answer this question, we start by:

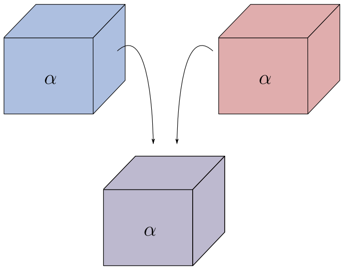
- take a dielectric consisting of n atoms per unit volume, with each atom possessing a nuclear charge q and a polarizability α, and
- place it between the plates of a parallel plate capacitor with plate area A and separation d:



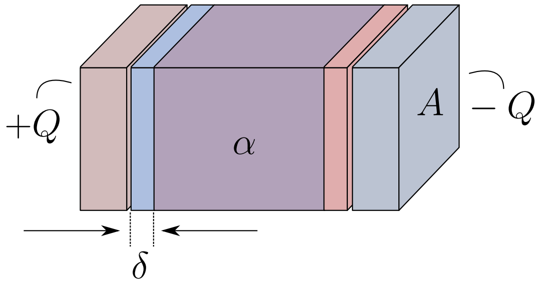
Next we place some charge $\pm Q$ on the plates of the capacitor. If we can determine the resulting voltage V across this dielectric filled capacitor, then we can take the ratio $\frac{Q}{V}$ to determine its capacitance.

The first thing we note is that depositing the charge on the capacitor plates will create an electric field E inside of the dielectric. The atoms in the dielectric respond to this electric field by polarizing, i.e. by displacing their clouds of negatively charged electrons with respect to their positively charge nuclei by some amount δ.

To see the macroscopic effect of this microscopic displacement δ, we want to first conceptually split the dielectric (purple) into its negative charges (blue) and positive charges (red):



The macroscopic effect of the microscopic displacement created by the electric field is to shift all the negative charge clouds a distance δ with respect to their associated positive nuclei:



The red and blue volumes mostly overlap, but at the end of the dielectric close to the positively charged plate we have exposed some negative charge, and vice versa. How large is this exposed charge Q_{exp} in the region near the positively charged plate? Well:

- The volume of the exposed region is given by the plate area A times the displacement δ.
- Since the dielectric contains n atoms per unit volume, this means the exposed volume contains $n\delta A$ clouds of negative charge.
- Since the charge of the nucleus of each atom is q, and the atoms are presumed to be electrically neutral, this means each cloud of negative charge has a charge $-q$.
- Therefore, the charge Q_{exp} is given by the equation:

$$Q_{exp} = -q\delta nA$$

So we find the effect of the electric field is to create some exposed charge $\pm Q_{eff}$ on the ends of the dieletric that are *opposite* in sign to the charge on the nearby plate. We can then think of the dielectric as acting to partially "screen" the charge on the capacitor plates, so that we effectively now have a charge $Q_{eff} = Q + Q_{exp} < Q$ on the positive plate of the capacitor, and a charge $-Q_{eff}$ likewise on the negative plate. This screening effect of dielectrics is their essential conceptual significance[4].

We can summarize the arguments above by stating that our dielectric-filled parallel plate capacitor holding a charge Q is equivalent to a *vacuum-filled* parallel plate capacitor holding a charge Q_{eff}. Therefore, the voltage V across the dielectric-filled capacitor is simply the effective charge Q_{eff} on the plates divided by the capacitance $C_o = \frac{\epsilon_o A}{d}$ of the same capacitor *without the dielectric*:

$$V = \frac{Q_{eff}}{C_o} = \frac{Q_{eff}}{\epsilon_o A/d} = \frac{Q + Q_{exp}}{\epsilon_o A/d} \tag{5}$$

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[4] In class a student asked what a *conductor* would do, and we saw that it would *completely* screen the charge on the capacitor plates, i.e. $Q_{eff} = -Q$ for a conductor. We might say then that the polarizability α of the metal atoms in a conductor is *infinite*. In other words a negative charge cloud can move *arbitrarily* far from its associated nucleus in response to an electric field, i.e. the charge carriers move *freely*.

From here we note that our expression for Q_{exp} contains the product $q\delta$. This product we recognize as the atom's dipole moment p, which in turn we recall is given by $\alpha E = \alpha V/d$. Making these substitutions we find

$$Q_{exp} = -\alpha V n A/d$$

Substituting the above equation into equation (5) we arrive at:

$$V = \frac{Q - \alpha V n A/d}{\epsilon_o A/d}$$

Finally, this equation can be rearranged to obtain:

$$\frac{Q}{V} = \frac{\left(1 + \frac{n\alpha}{\epsilon_o}\right)\epsilon_o A}{d}$$

The ratio on the left is ratio of the charge Q placed on the dielectric-filled capacitor to the voltage V across it. This ratio is then the capacitance C of the dielectric filled capacitor. By comparison with equation (2), we find that the relative permeability ϵ_r of the dielectric filling the capacitor is given by

$$\epsilon_r = 1 + \alpha n/\epsilon_o \tag{6}$$

This equation is the connection between the macroscopic relative permittivity ϵ_r of a dielectric and the microscopic polarizability α of the atoms making up the dielectic. Qualitatively, the equation tells us that, by inserting a material containing atoms of a high polarizability α between the plates of a capacitor, we can greatly increase its capacitance. We can further amplify this effect by increasing the density n of atoms in the material.