302L - Lecture 6
Feb. 7rd

Conductors and Gauss' Law
Serway 15.6

Main points:

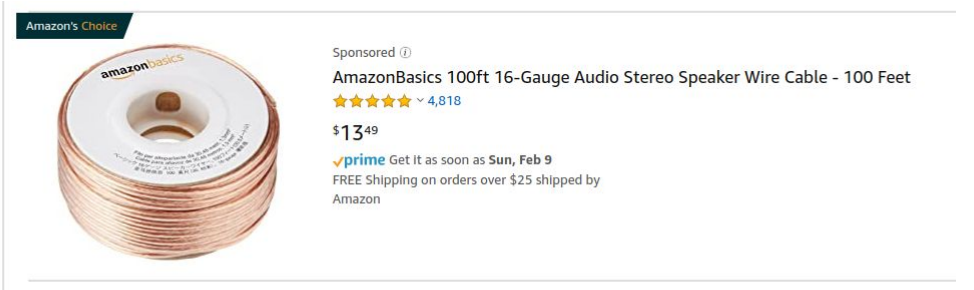
- Electric field is zero inside of a conductor
- The inside of a conductor is electrically neutral (charge *density* is zero)
- Any charge imbalance occurs at the surface
- The electric field vector at a point on the surface has no component tangent to the surface
- Surface charge density is at regions of high curvature (e.g. sharp points and edges)
- Strong forces acting normal to the surface prevent charge from escaping.
    - Ways of removing electrons from a metal surface: photo-emission, thermionic emission, field emission.

Conductors revisited

Electrical conductors contain freely moving charged particles.

examples

- copper (speaker wire)
  - *electrons* carry charge (ions / nuclei are stuck)
- salt water
  - *ions* carry charge
- plasma (ionized gas)
  - electrons and ions carry charge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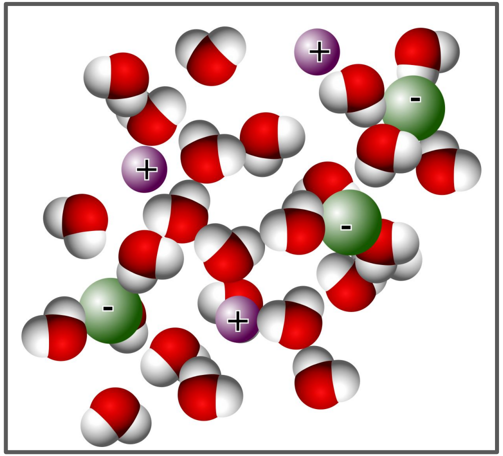
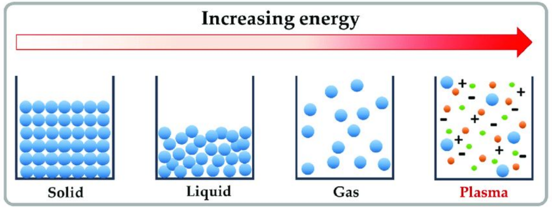




“star in a jar”

Mycotoxin Decontamination of Food: Cold Atmospheric Pressure Plasma versus “Classic” Decontamination

Conductors revisited


La Croix Sparkling Water, 12 Ounce (48 Cans) (Grapefruit)

★★★★☆ 55

$33.15 ✓prime

17 used and new from $32.00

Electrical c
moving ch

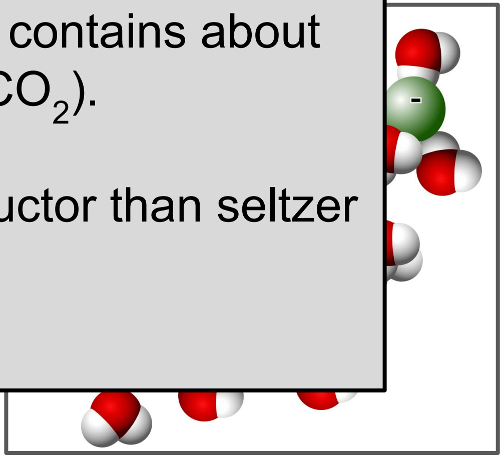
examples

- coppe
  - el
    nu
- saltwa
  - io

Instapoll question:

A liter of tap water contains roughly a 100mg of dissolved salt (NaCl) while a liter of seltzer water contains about 1000mg of carbon dioxide (CO_2).

Why is tap water a better electrical conductor than seltzer water?



Conductors and Electrostatic equilibrium

- In equilibrium, the net force on all particles are zero
- For conductors, the charged particles move *freely* so the only force they experience is from the electric *field* at their location
- Therefore, the electric field in a conductor is zero

$$\vec{F} = 0$$

$$\vec{F} = q\vec{E}$$

$$\boxed{\vec{E} = 0}$$

Conductors, charge, and Gauss' Law - charge inside

- Is it possible for there to be a *charge imbalance* inside of the conductor while still maintaining a zero electric field?
- Charge imbalance: region containing more / less positive charge than negative.

$$\vec{E}(\vec{x}) = 0 \Rightarrow \rho(\vec{x}) = 0?$$

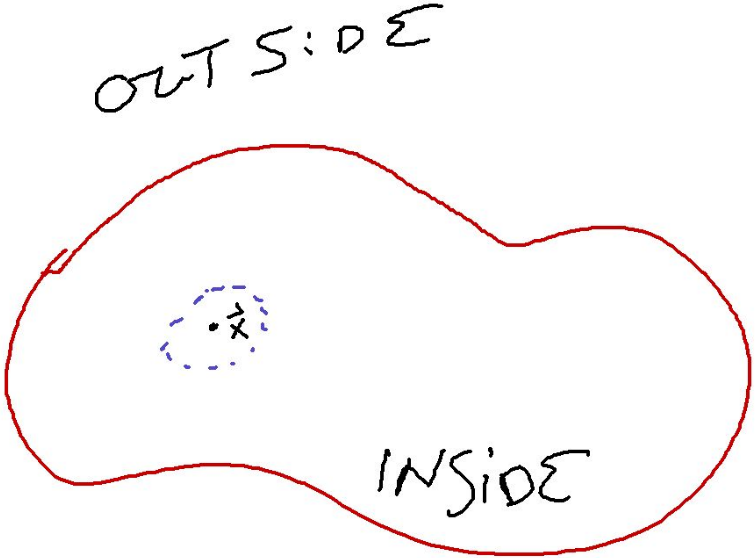
"volume charge density" $\rho(\mathbf{x})$:

- way of describing the charge imbalance at a location **x**
- the net charge ΔQ in a small region of volume ΔV around the point **x** is given by $\Delta Q = \rho(\mathbf{x})\Delta V$
- e.g. a cube of side length *a* with an evenly distributed charge Q has a charge density $\rho = Q/a^3$

Conductors, charge, and Gauss' Law - charge inside

To see that there there is no charge imbalance *inside* a conductor :

- Draw a small Gaussian surface of volume ΔV around any point **x** inside the conductor (red walls).
- Because **E** = 0 everywhere on Gaussian surface, the flux Φ is zero and so the charge *enclosed* ΔQ=ρ(**x**)ΔV must also be zero.
- Works for any surface we can draw, so charge density ρ(**x**) is zero everywhere inside.

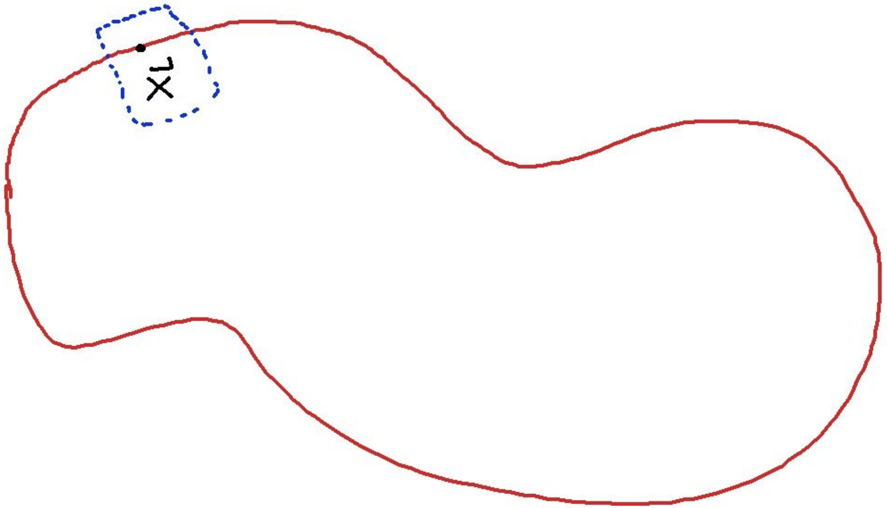


$$\boxed{\rho(\vec{x}) = 0 \text{ inside conductor}}$$

Conductors, charge, and Gauss' Law - surface charge

What about points **x** *on the surface* of the conductor?

- Any Gaussian surface around the point must have points *outside* the conductor!
- No more guarantee that **E** is zero everywhere on Gaussian surface



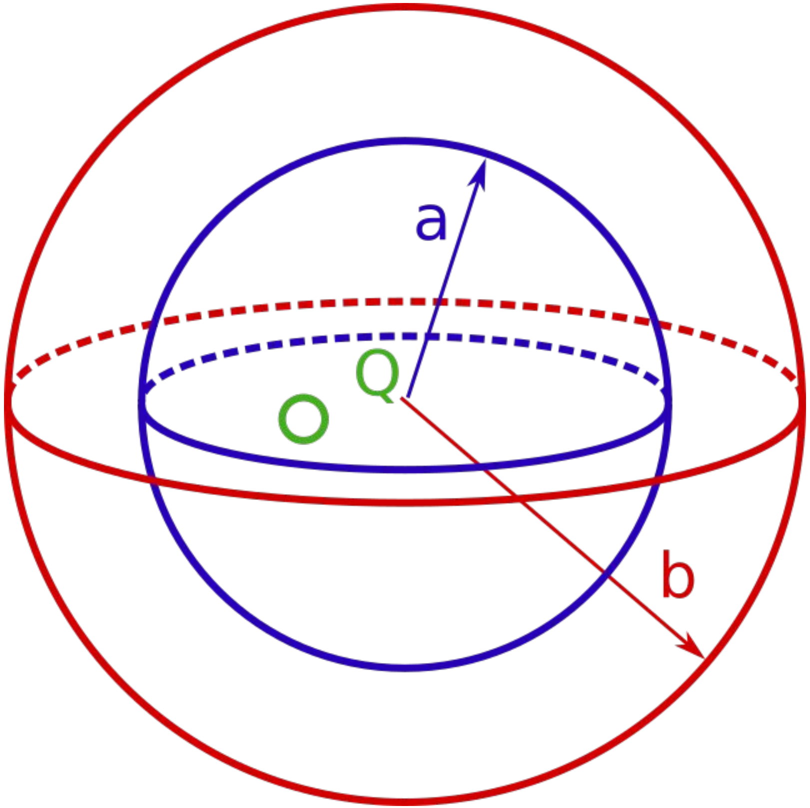
Any charge imbalance on a conductor can only occur at its surface

Conductors, charge, and electric field: practice problem

Suppose we place an object with charge Q inside the hollow portion of an uncharged conductive spherical shell.

What is the charge induced on the *inner surface*?

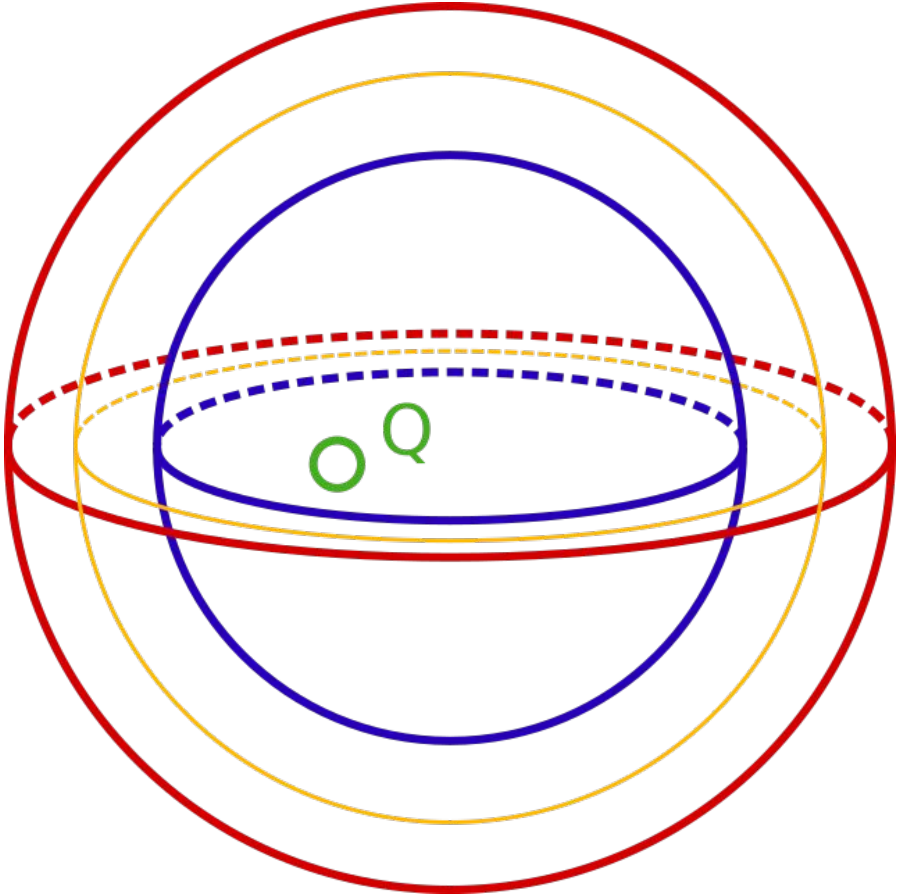
What is the charge induced on the *outer surface*?



Conductors, charge, and electric field: practice problem

Solution:

- Construct a **closed Gaussian surface** that lies entirely *inside* the conductive shell.
- The electric field is zero everywhere on the **Gaussian surface** so the charge enclosed must be zero

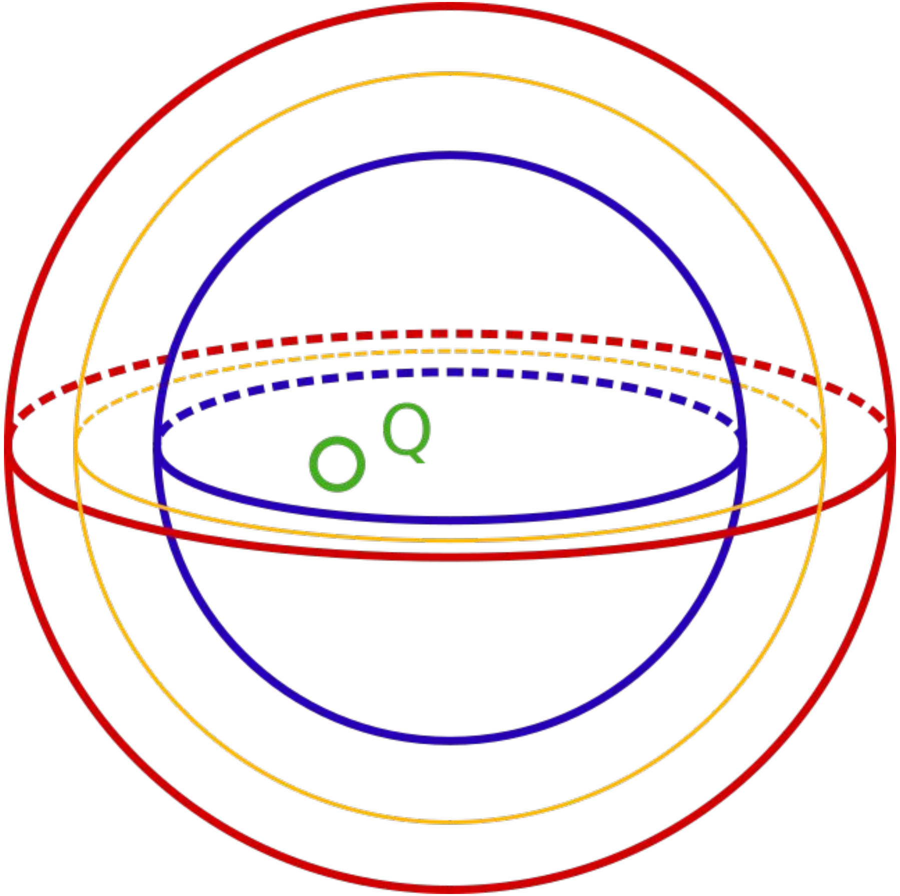


Conductors, charge, and electric field: practice problem

Solution:

- The charge enclosed is the sum of
  - the charge Q of the object inside the hollow part of the sphere, plus
  - whatever the charge Q_{in} is on the inside surface of the sphere
- The two charges must sum to zero so:

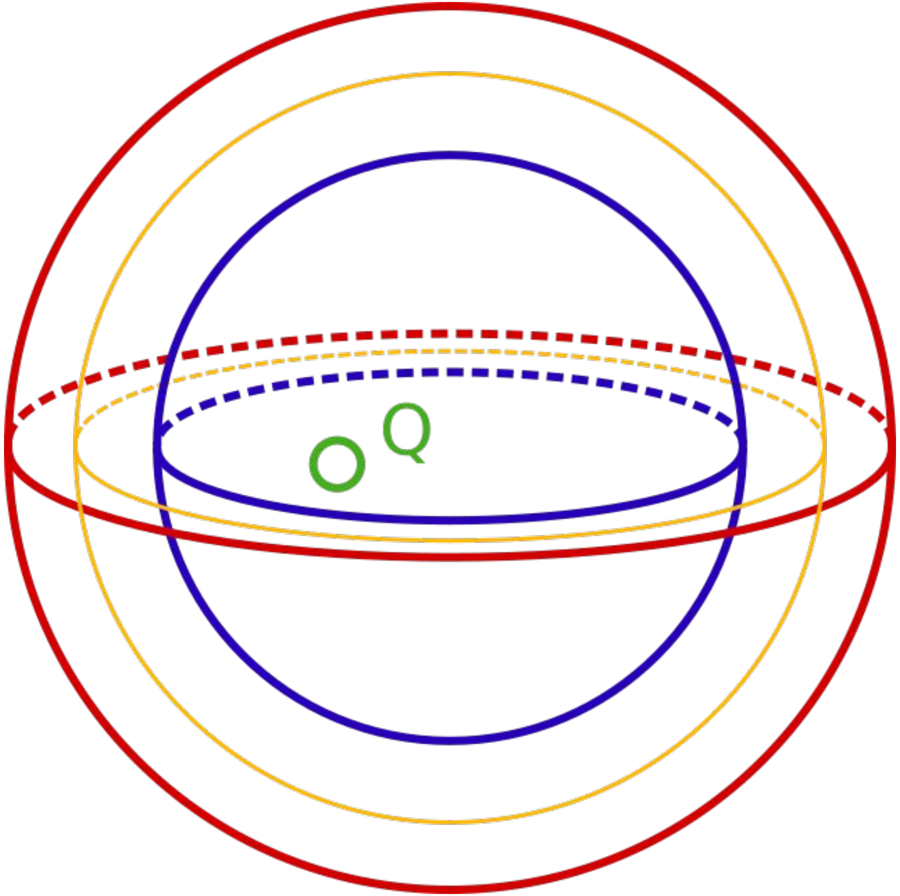
$$Q_{in} = -Q$$



Conductors, charge, and electric field: practice problem

Solution:

- And so what about the charge Q_{out} on the outside surface?
- Instapoll: what is Q_{out}? Why?

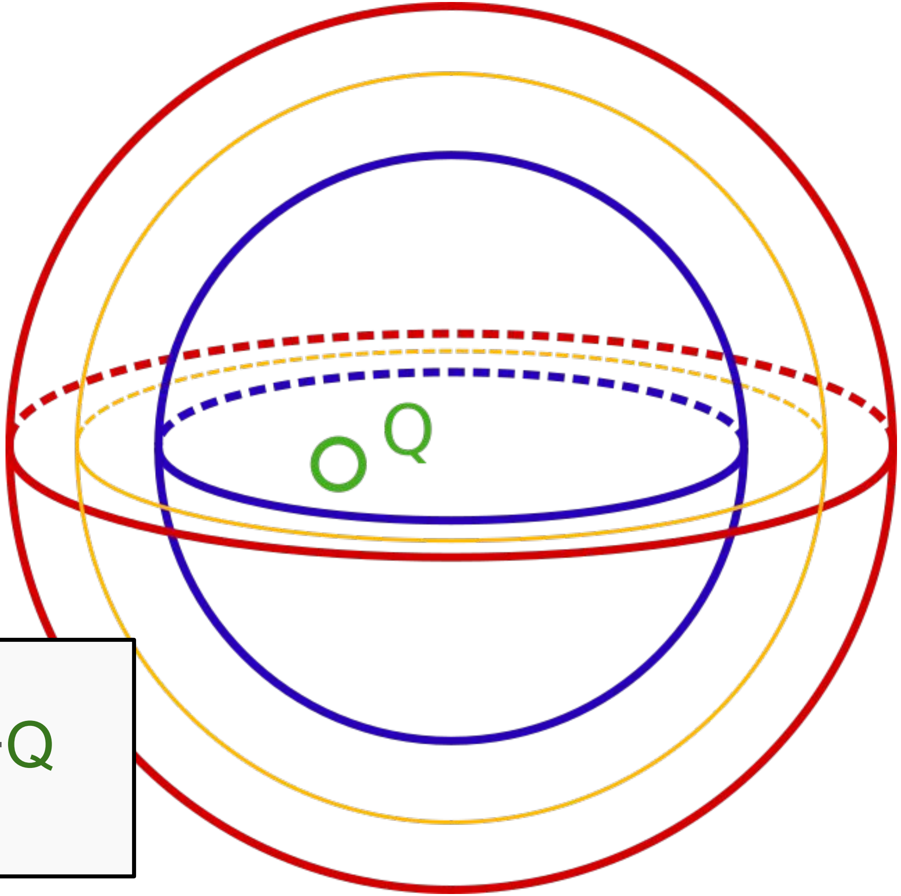


Conductors, charge, and electric field: practice problem

Solution:

- And so the charge Q_{out} on the outside surface?
- Instapoll: what is Q_{out}? Why?
- Since the sphere was initially *uncharged*, we can conclude

$$Q_{in} + Q_{out} = 0, \text{ or } Q_{out} = -Q_{in} = +Q$$

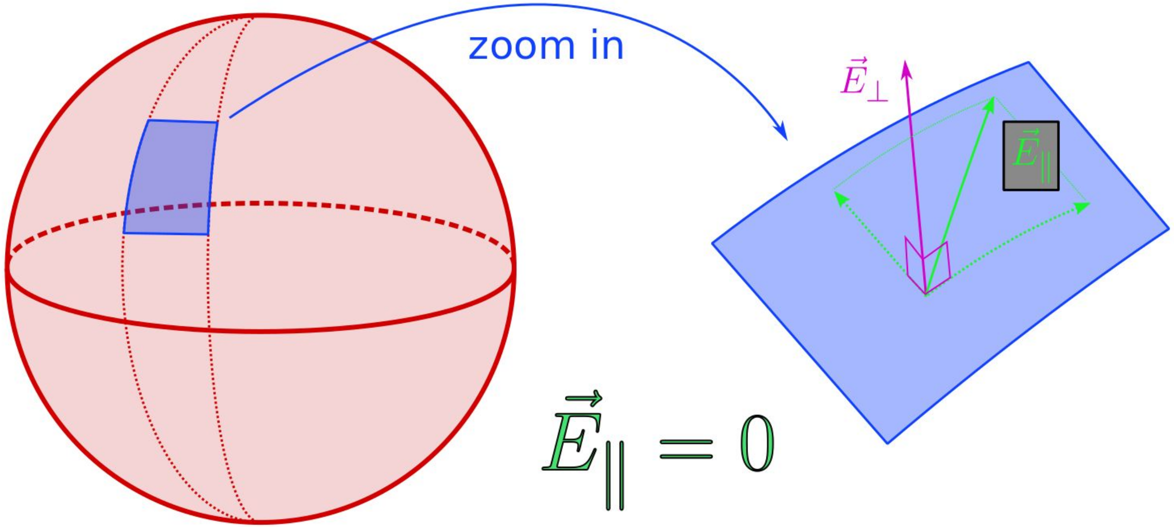


Conductors: surface electric field

What is the electric field ***on*** the surface?

Because the charge can move freely along the surface, equilibrium requires the electric field to point ***normal*** to the surface.

If it *didn't*, then charge would flow along the surface until it *did*.



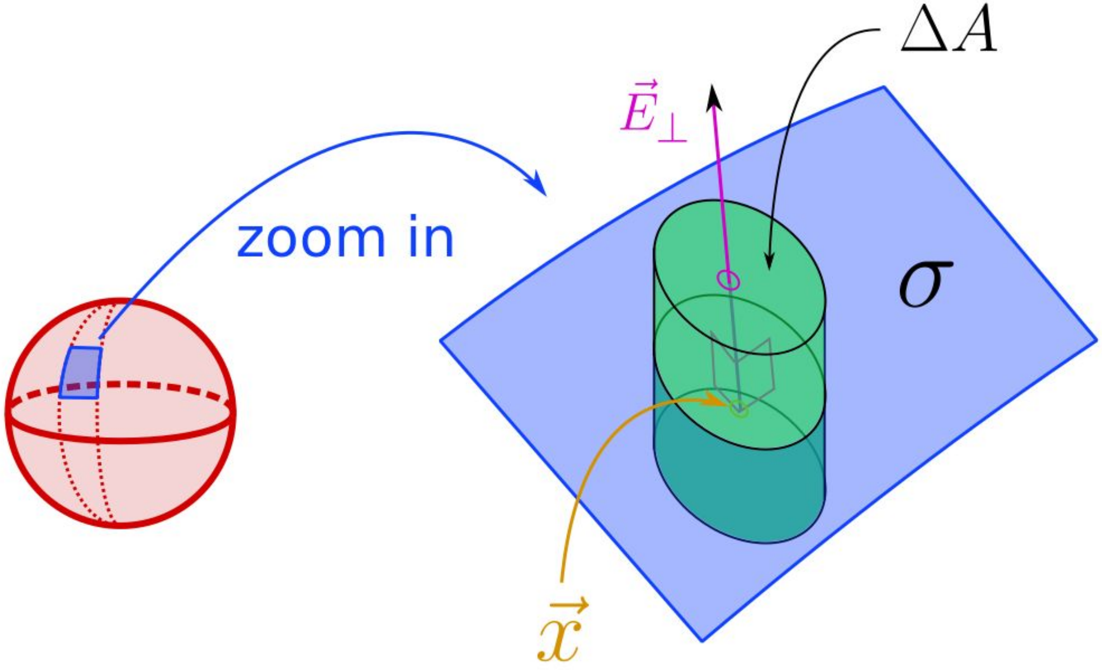
$$\vec{E}_{||} = 0$$

Conductors: surface electric field

What is the strength $E_\perp$ of the electric field at a point **x** on the surface, if the surface charge density at **x** is σ ?

Procedure:

- Enclose the point **x** with a stubby Gaussian cylinder of cross sectional area ΔA, with its axis parallel to the surface.

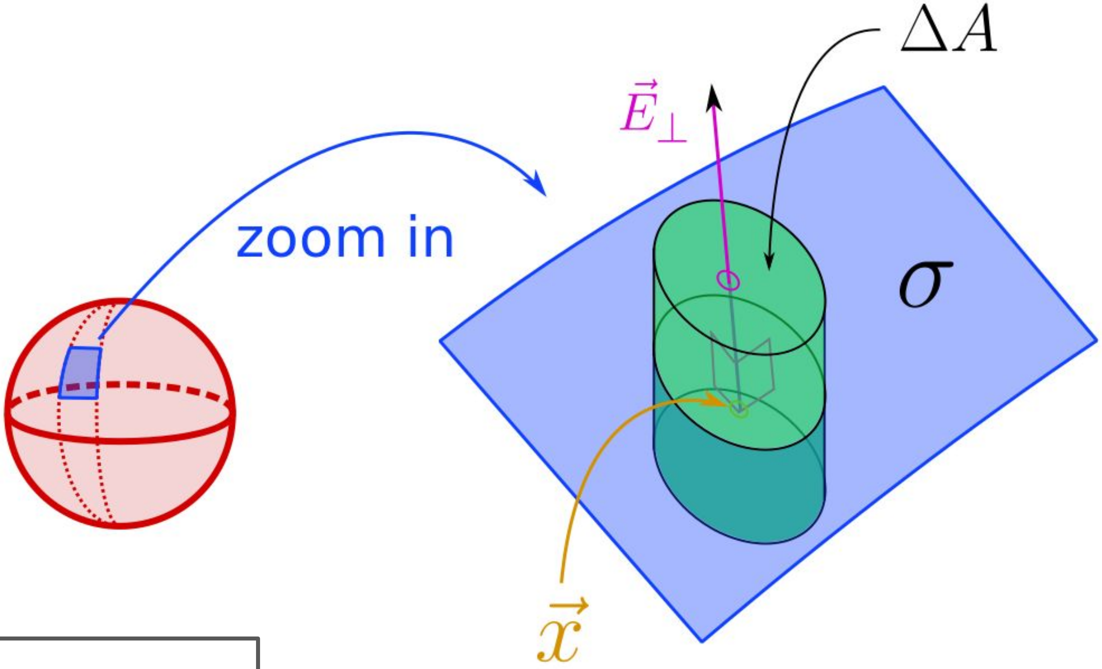


Conductors: surface electric field

Procedure (continued):

- Flux Φ through Gaussian surface is zero except through top end cap (why?)
- Flux Φ through top endcap is $\Phi = E_{\perp}$ □ ΔA
- Flux Φ by Gauss' Law is $\Phi = \sigma \Delta A / \epsilon_o$
- Therefore:

$$\boxed{E_{\perp} = \sigma / \epsilon_o}$$



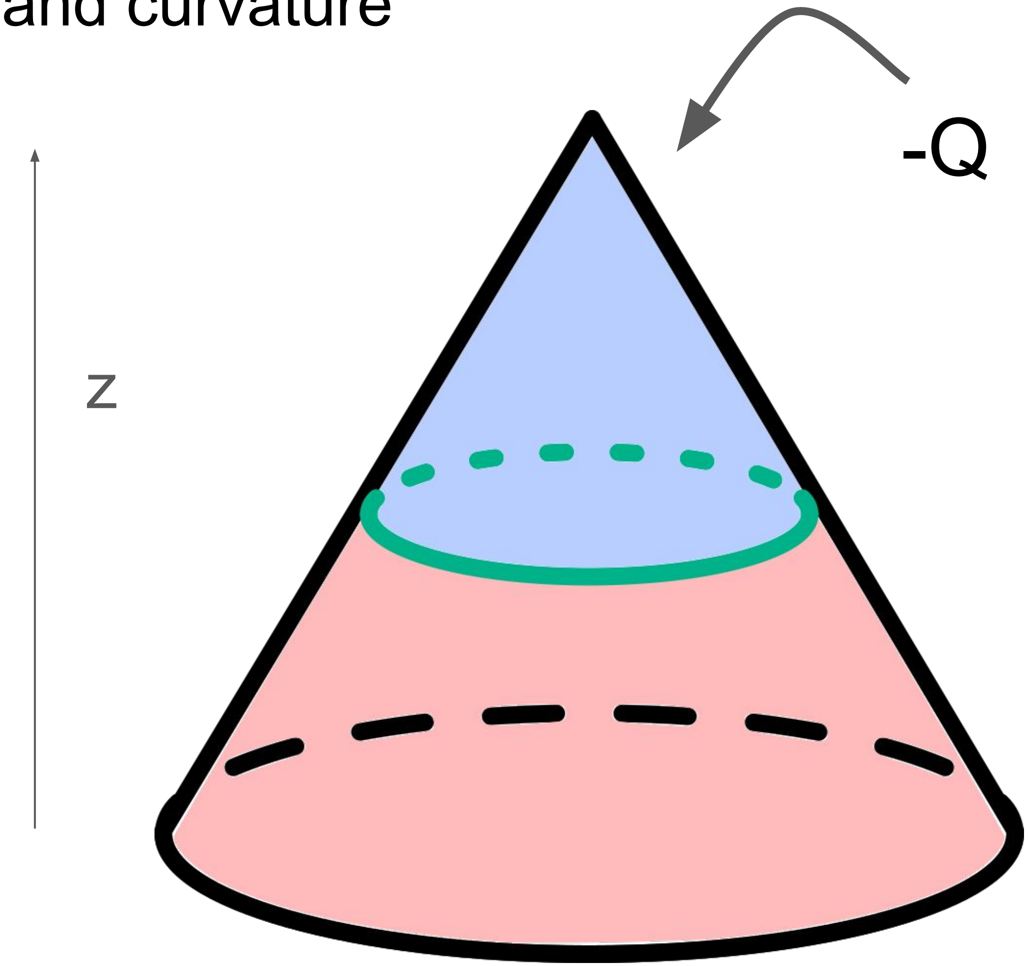
Conductors: surface charge and curvature

Let's convince ourselves that charge will tend to accumulate in regions of high curve, e.g. pointy tip and sharp edges

As an example:

Place a certain quantity of negative charge -Q on a cone. How will this charge distribute itself?

i.e. what is σ(z)?



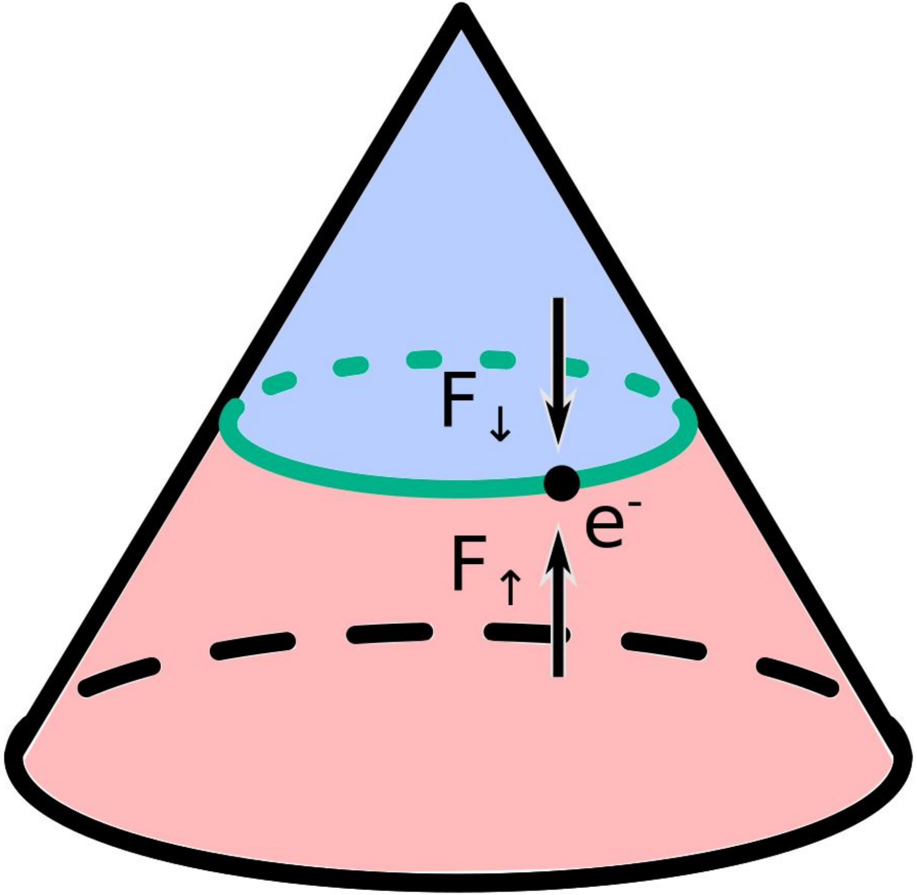
Conductors: surface charge and curvature

(continued)

Consider an electron on the green ring (halfway down).

In equilibrium, the forces $F_\uparrow$ pushing the electron up towards the tip equal the forces $F_\downarrow$ pushing the electron down.

To get canceling forces on the the electron, the amount of charge on the blue strip must then equal the charge on the red strip.



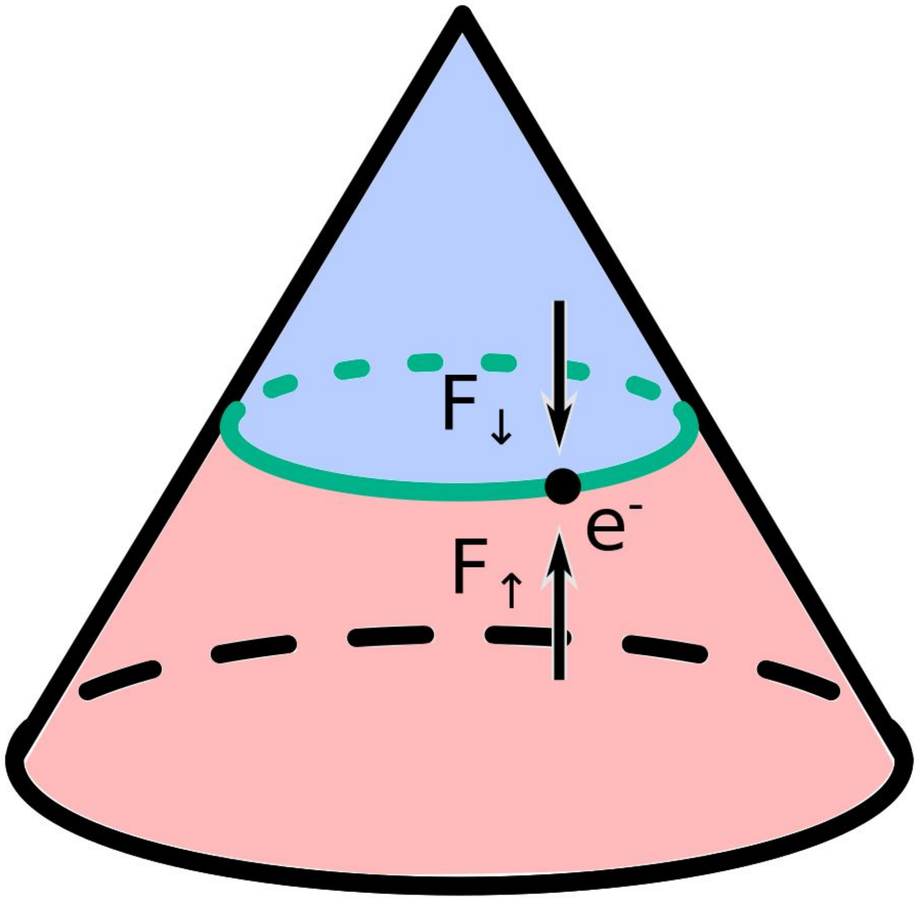
Conductors: surface charge and curvature

(continued)

There is less area on the blue strip than the red to hold the charge, so its (surface) charge *density* | σ | must be larger!

In general we conclude:

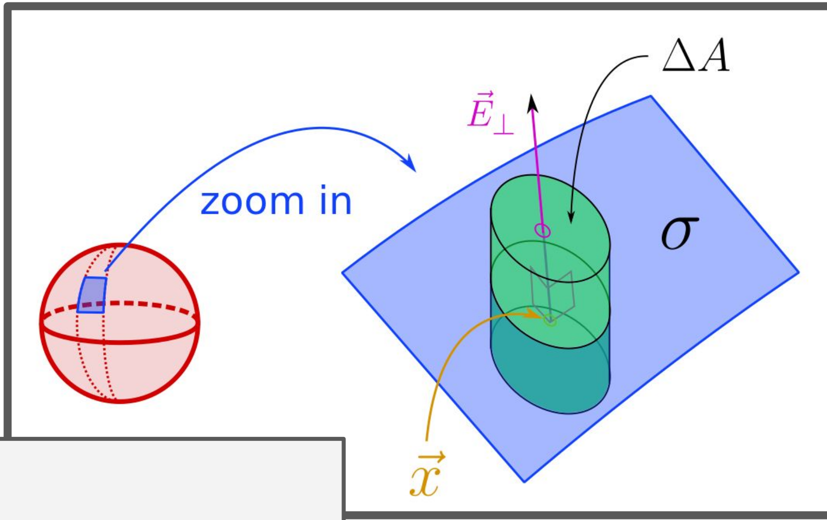
On a conductor, surface charge accumulates in regions of tighter curvature.



Conductors: “normal force”

If the electric field points out of the surface, then there is a force pushing surface charge *away* from the surface.

why isn’t the surface charge accelerated *off* the surface?



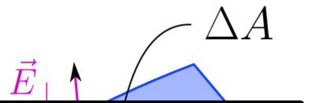
$$E_\perp = \sigma / \epsilon_o$$

Conductors: “normal force”

If the electric field points out of the surface, then there is a force pushing surface charge *away* from the surface.

If ~~there are supposedly no other forces exerted on our charged particles~~, then why isn’t the surface charge pushed/accelerated *off* the surface?

$$E_\perp = \sigma / \epsilon_0$$



There **are** strong forces exerted by the material preventing charged particles from escaping.

- forces are *chemical* in nature
- analogies:

Conductors: “normal force”

If the electric field points out of the surface, then there is a force pushing surface charge *away* from the surface.

If ~~there are supposedly no other forces exerted on our charged particles~~, then why isn’t the surface charge pushed/accelerated *off* the surface?

$$E_\perp = \sigma / \epsilon_0$$

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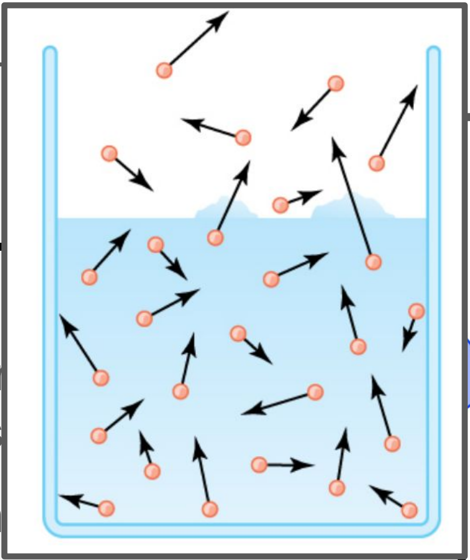
preventing charged particles

- forces are *chemical* in n
- analogies:
  - *latent heat* of evaporation: energy required to extract molecules from surface of liquid



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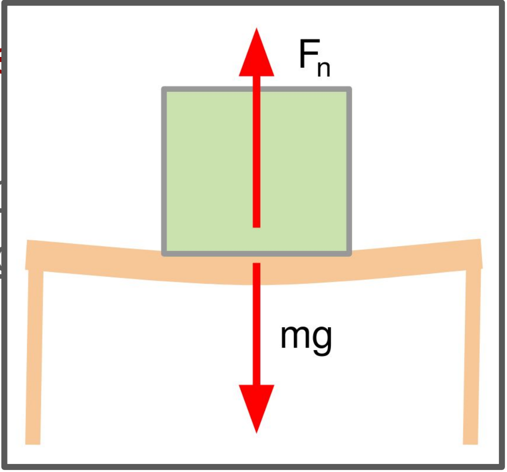
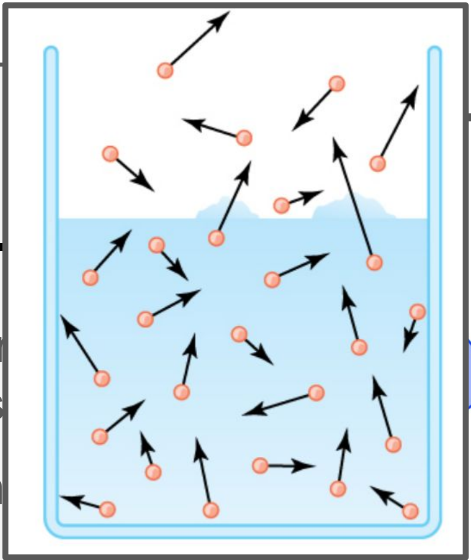


$E_\perp = \sigma / \epsilon_0$

There **are** strong forces exer preventing charged particles

- forces are *chemical* in n
- analogies:
    - *latent heat* of evaporation: energy required to extract molecules from surface of liquid
    - the *normal force* that prevents your foot from falling through the ground



Conductors: “normal force”

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How do we *remove* charge from the surface?

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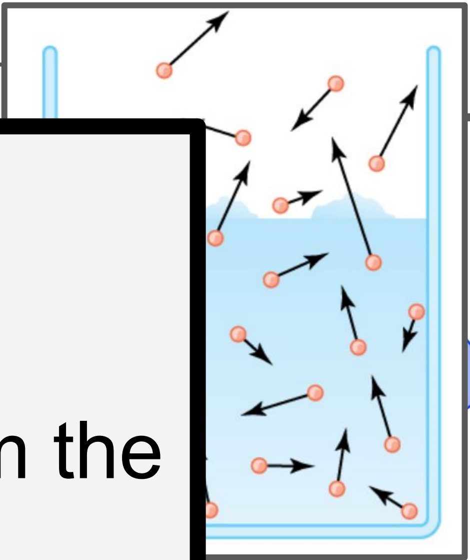
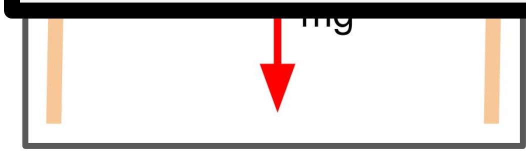
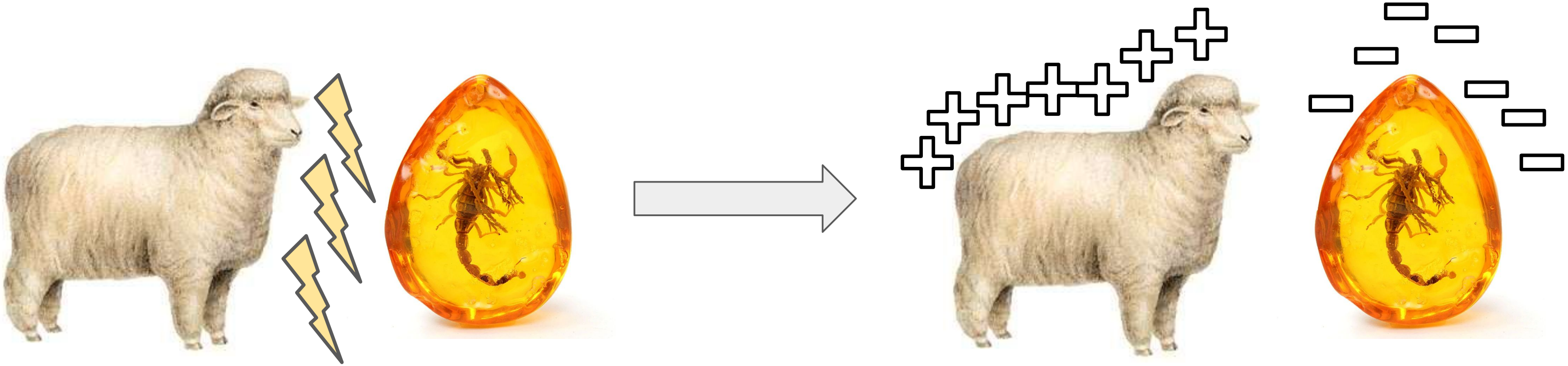
from falling through the ground





Emission of charged particles from conductors: *triboelectricity*

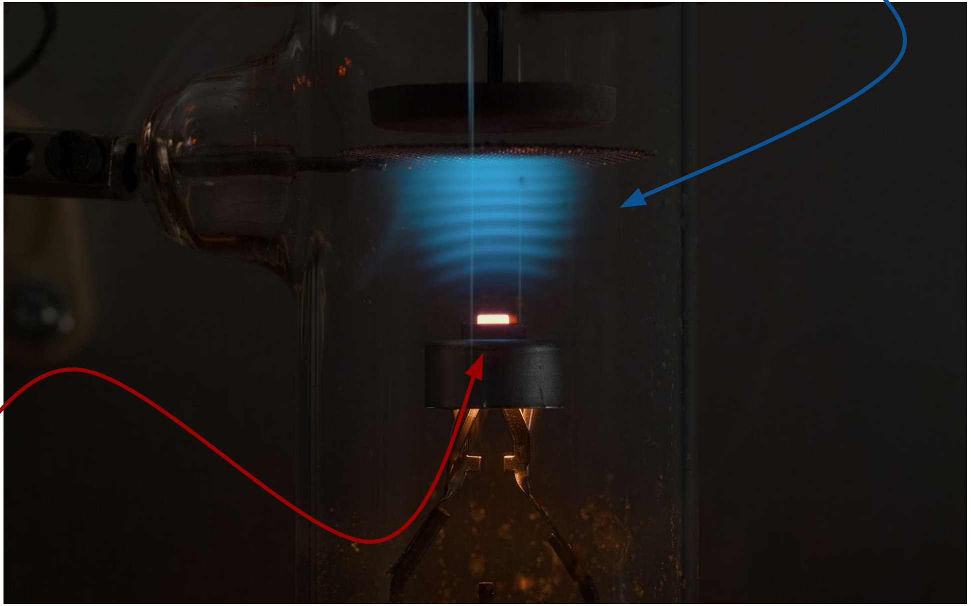
We already learned that *friction* between materials can remove charge:

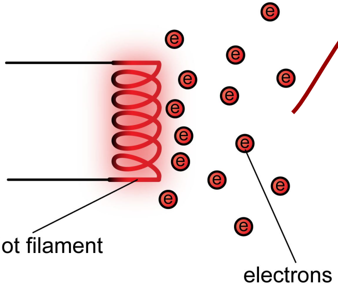


Emission of charged particles from conductors: *thermionic emission*

- By *heating* a metal surface, we can strongly agitate the electrons
- Some electrons gain enough momentum to overcome confining force
- Electrons essentially ***"boil"*** off the surface

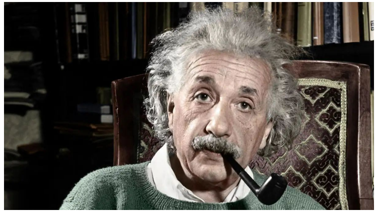
emitted electrons excite gas, causing gas to glow

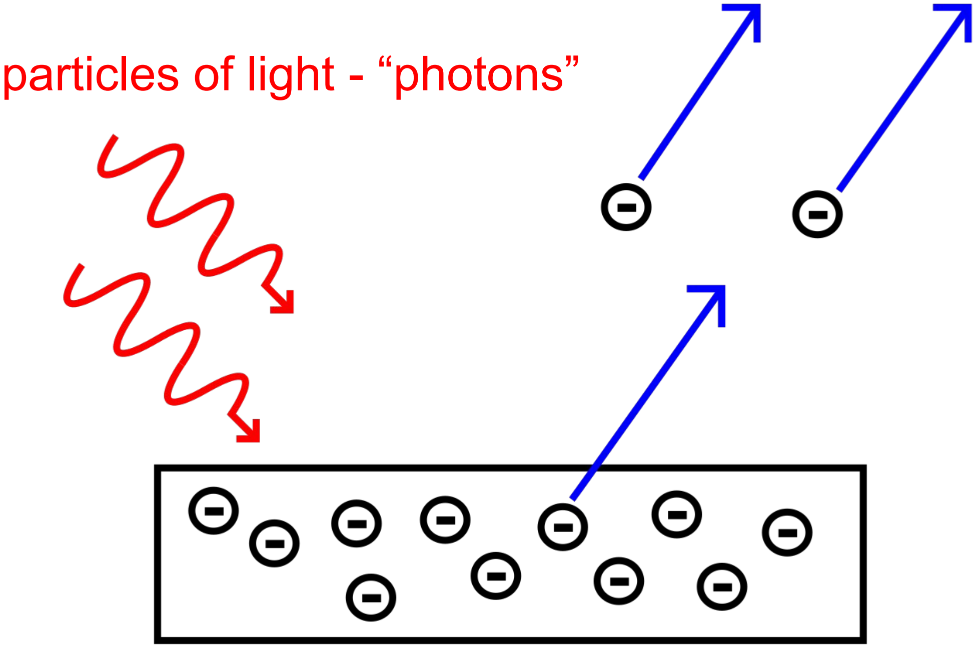




Emission of charged particles from conductors: *photo-emission*

- *photo-electric effect* - Short wavelength light (e.g. blue or ultra-violet light) causes current to flow across an air gap
- light is composed of particles that give their entire energy to a single electron, boosting it out of the surface.

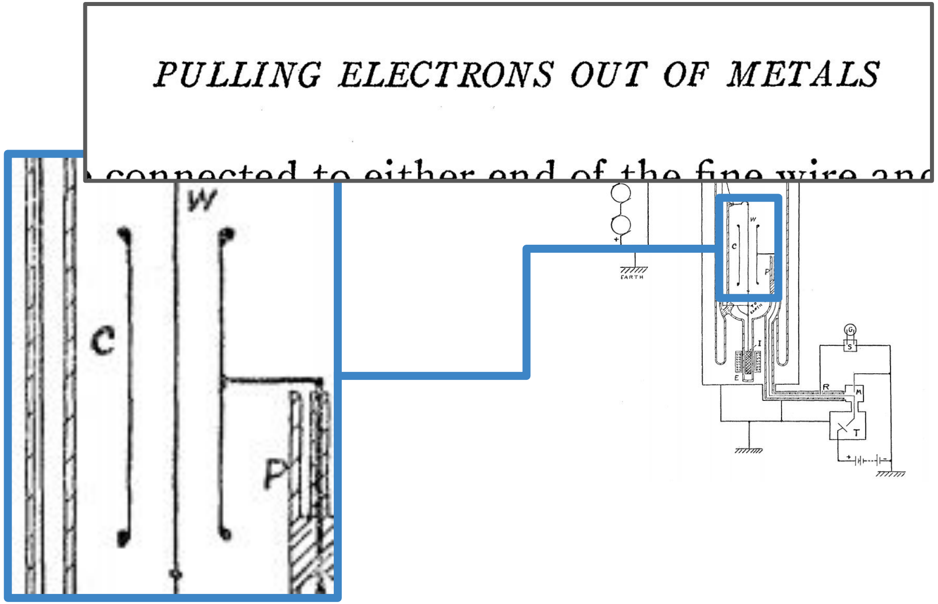




Emission of charged particles: *field-emission*

- By putting a lot of charge on a conductor, can overcome confining force

$$\boxed{E_\perp = \sigma / \epsilon_o}$$



PULLING ELECTRONS OUT OF METALS

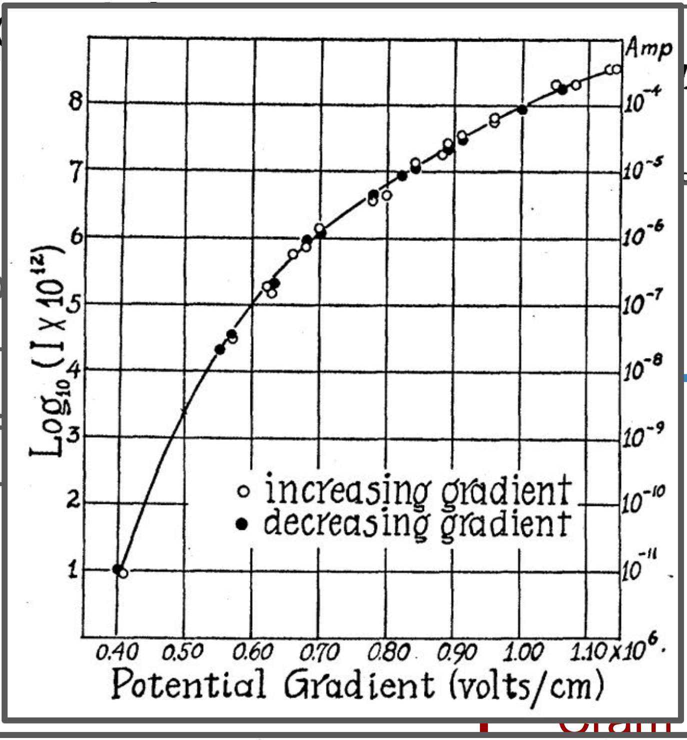
connected to either end of the fine wire an

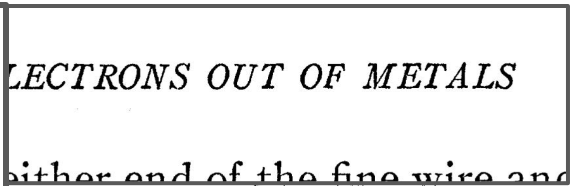
Cram negative charge onto wire (W)

Emission of charged p
field-emission

- By putting a lot of charg a conductor, can overco confining force

$E_\perp$ =




LECTRONS OUT OF METALS

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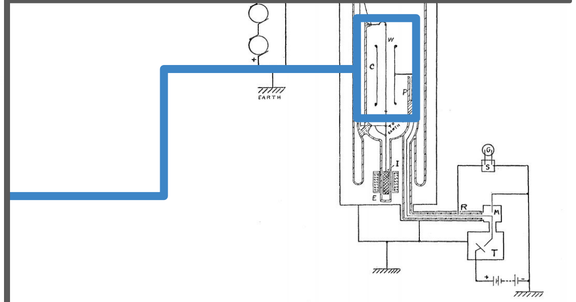
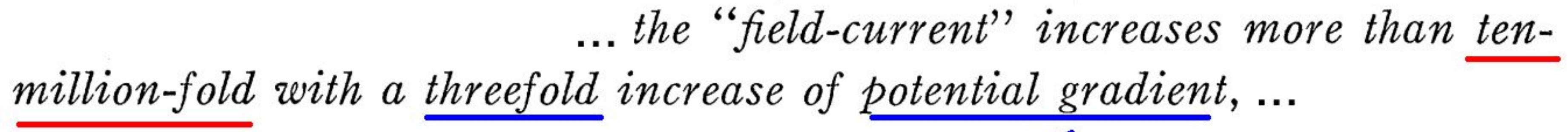


negative


… the "field-current" increases more than ten-million-fold with a threefold increase of potential gradient, …

i.e. charge