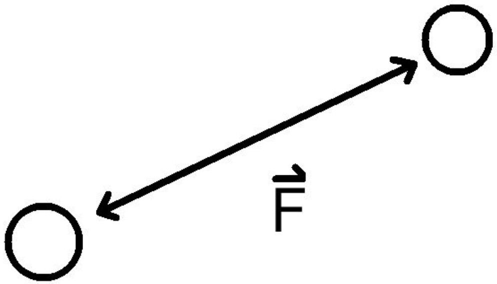
302L - Lecture 3

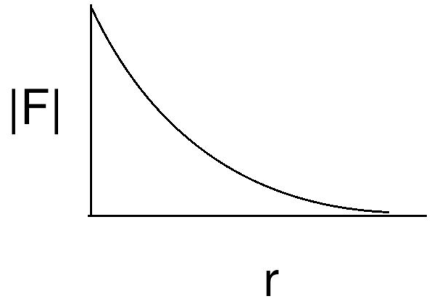
Coulomb's Law + Electric Field
(Serway sections 15.3-15.4)

Electrical force - recap

From the previous lecture we can summarize a few things about the force applied by charged objects on one another:

- The force applied between two charged objects is directed along the line joining them (this is simply what we *mean* by attraction/repulsion).
  - Newton's 2nd law + conservation of angular momentum
- The magnitude of the force decreases with increasing distance (r).
- <u>What is the dependence</u> |F(r)| ???
  - energy conservation demands no direction dependence on force magnitude



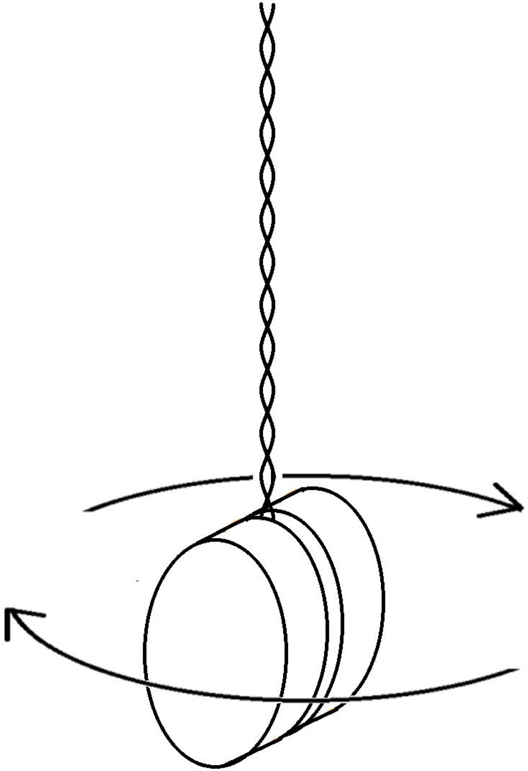


Coulomb’s torsion balance experiment

Authoritative determination of distance dependence for electric force.

Coulomb was an expert on *torsion* experiments.

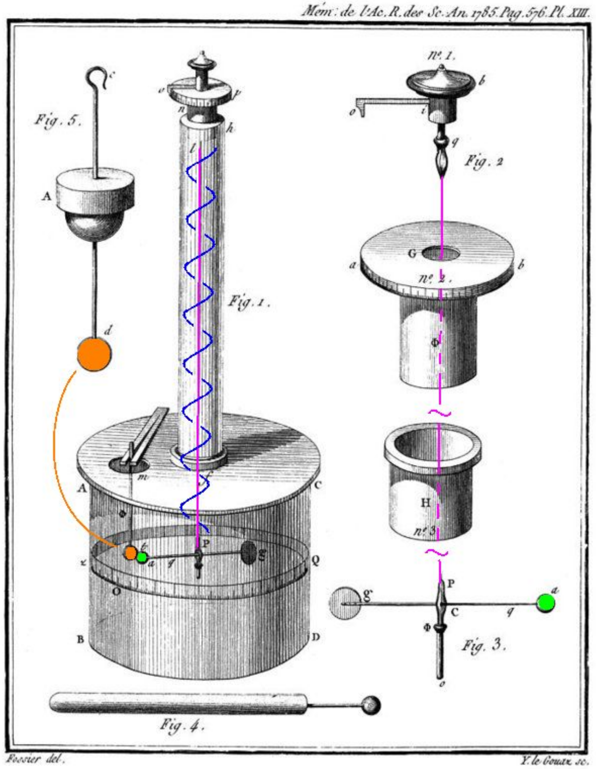
torsion relates to the generation of torque ($\propto$ force) by the *twisting* of an object (think of an unspooled yo-yo string twisting/untwisting)



Coulomb's torsion experiment - apparatus

A conducting **sphere** is attached to a narrow **wire** that can be **twisted**.

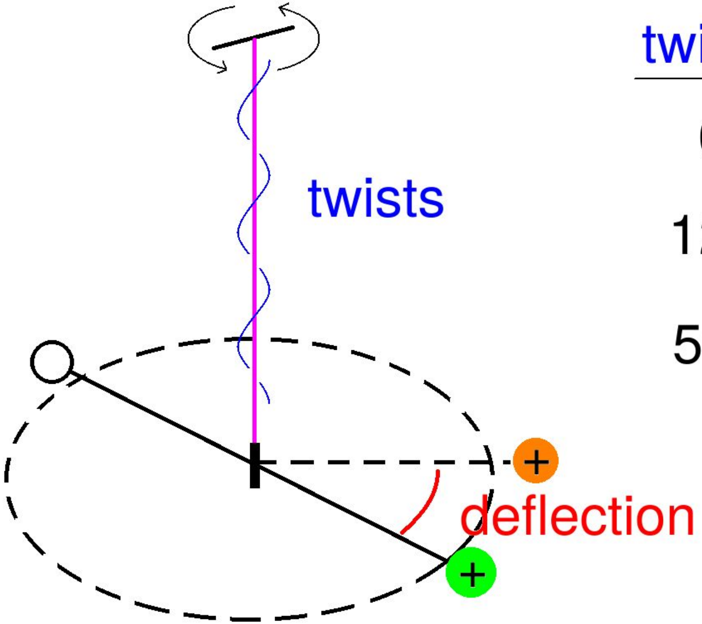
A second stationary conducting **sphere** is charged and then the two spheres are brought into contact, at which point they repel.



Coulomb's experiment - diagram and **full** results

The sphere attached to the wire comes to rest at a certain angular deflection where the torsion force balances the electric repulsion.

The wire is then twisted in the direction pulling the spheres together, and the change in deflection in recorded.



twist	deflection
0^o	36^o
126^o	18^o
567^o	8.5^o

Coulomb’s experiment - discussion

Torsion device extremely sensitive - 8 *millionths* of an ounce to twist 360°!

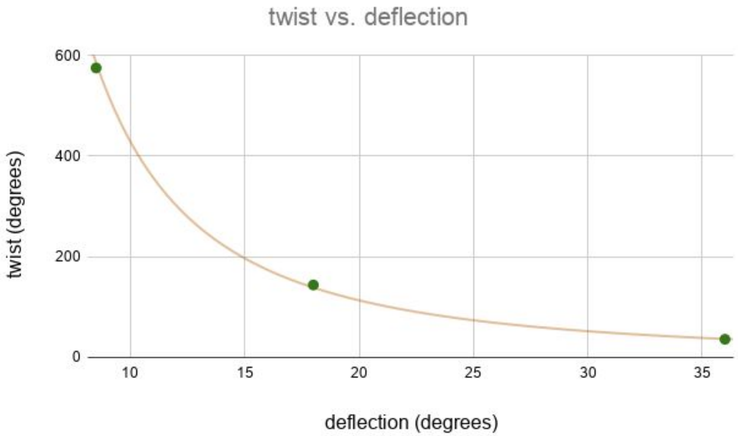
These three data points only data Coulomb offered as proof!

Regression gives $F(r) \propto r^{-1.92}$

Heavy theoretical bias towards Newtonian inverse square law

Inconsistencies in Coulomb’s account - “...it has to be remarked that...the torsion should never be more than 300° ”. What about third data point?!

Did he fudge data? (See articles in Canvas Files).



Main points: Coulomb's Law

The force **F** applied on a particle with charge q_1 by another particle of charge q_2 is given by:

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_o r^2}\hat{r}_{21}$$

where:

- ϵ_o is a constant called the permittivity of free space. In MKS units we have $\epsilon_o = 8.8541878128(13)\times10^{-12}\ C^2 \cdot N^{-1} \cdot m^{-2}$
- *r* is the distance between the two particles, and $\hat{r}_{21}$ is a vector of unit length pointing ***from*** q_2 ***to*** q_1
- The MKS unit of electric charge is the Coulomb (abbrev. C) and is defined such that Coulomb's force law (above) holds

Main points: Coulomb's Law

The fo q_2 is
given

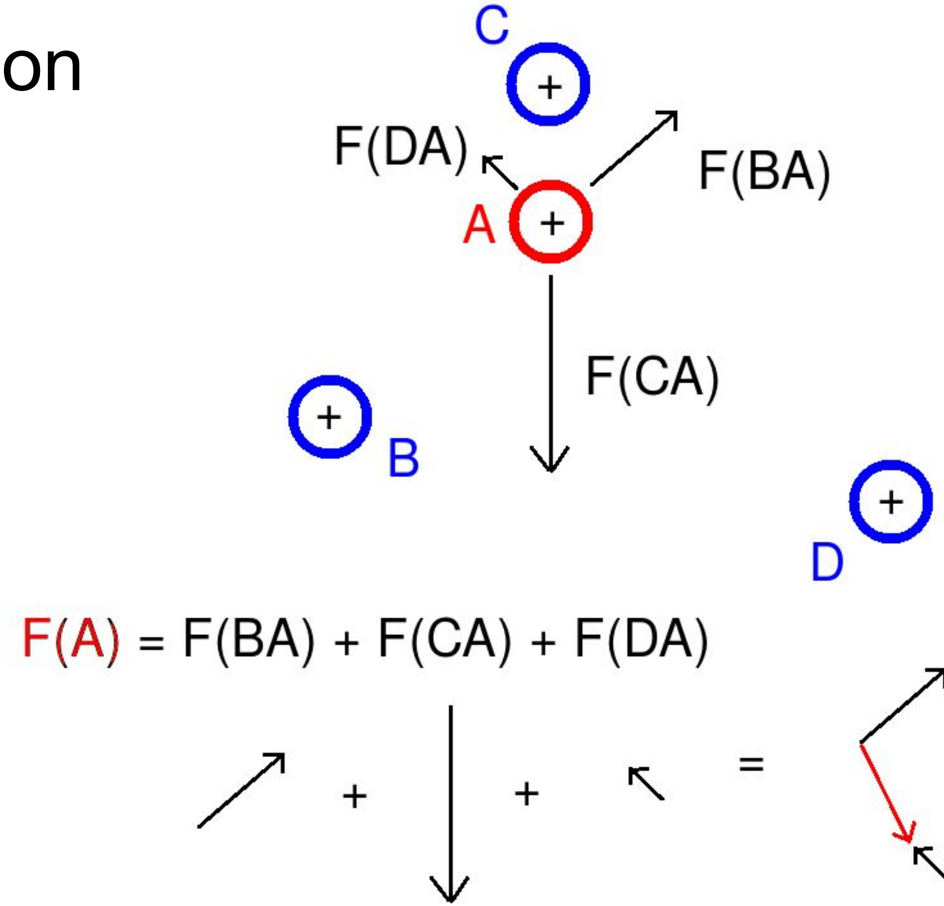
where

- ϵ_o e
  ϵ_o
- *r* i th
  po
- Th d
  such that Coulomb's force law (previous slide) holds

Ok but what if we have more than two particles? What is the force then?

Main point: Superposition

Superposition principle: The force exerted by one charged particle on another is *unaffected* by the presence of other charges. To determine the net electric force **F(A)** on one particle *A* applied by a collection of other particles *B*, *C*, *D*, …, we simply compute the (vector!) forces **F(***BA***)** + **F(***CA***)** + **F(***DA***)** + … applied by *B*, *C*, *D*, … on *A* and add them together.



Main point: Electric Field

Instead of working directly in terms of the *force* applied on some charge *A* by a collection of other charges (*B*, *C*, *D*, …) , it is often convenient to work instead with the *electric field* $\mathbf{E}(\mathbf{x})$ associated with the collection (*B*, *C*, *D*, …).

The electric field $\mathbf{E}(\mathbf{x})$ is a *function* that takes in a point in space **x** and returns a *vector*, that, when multiplied by the charge q_A of some particle *A*, equals the *force* on particle *A* applied by the collection (*B*, *C*, *D*, …) if particle *A* is located at the point **x**, i.e.

$$\mathbf{F}_A = q_A\, \mathbf{E}(\mathbf{x}) \qquad \text{(where particle } A \text{ is located at } \mathbf{x}\text{)}$$

place-holder?

↓

photon?

Electric field side note: vector fields

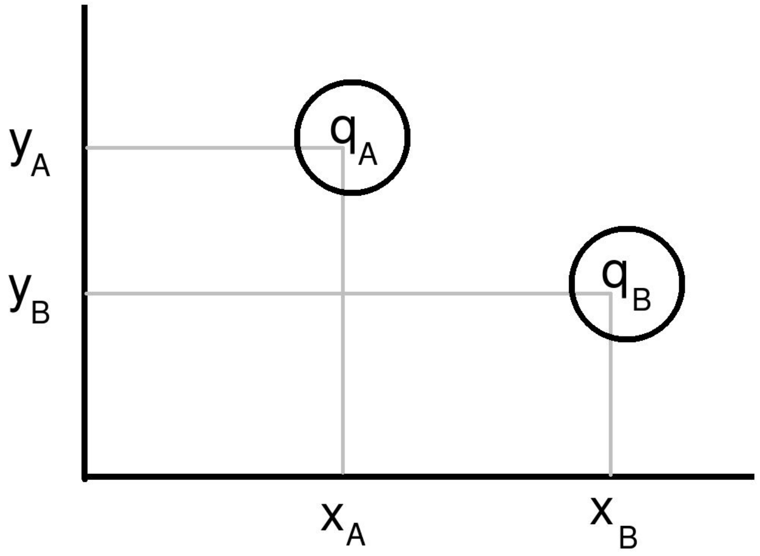
Keep in mind that, as I've defined it, an electric field **E** is <u>*not*</u> a vector, but rather a *function* that takes in a point **x** in space and *returns* a vector **E**(**x**).

Such objects are termed *vector fields*.

Electric field example: single point charge

If our collection of charges consists of a single particle (*B*) of charge q_B located at a point $\mathbf{x}_B=(x_B,y_B)$, then we can work backwards from Coulomb's force law to determine the electric field.

i.e. if we place a second particle *A* of charge q_A at a point $\mathbf{x}_A=(x_A,y_A)$ then particle *A* experiences a force

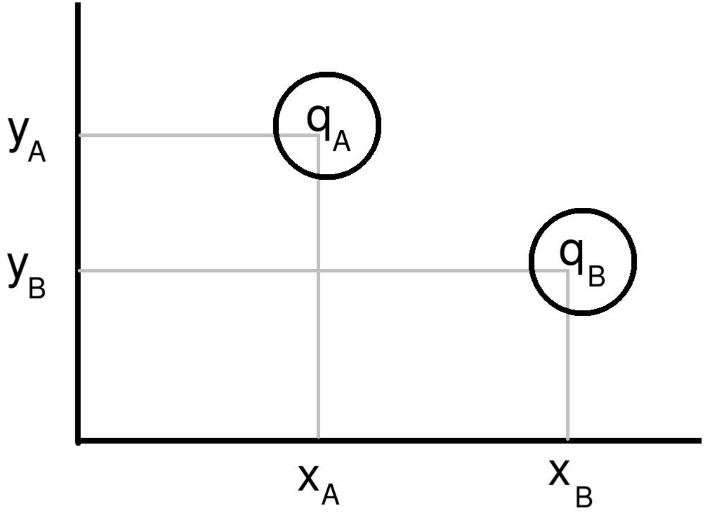


$$\vec{F}_A = \frac{q_A q_B}{4\pi\epsilon_o|\vec{x}_A - \vec{x}_B|^2} \cdot \frac{\vec{x}_A - \vec{x}_B}{|\vec{x}_A - \vec{x}_B|}$$

Electric field example: single point charge

By definition, this is equal to q_A multiplied by the electric field $\mathbf{E}(\mathbf{x}_A)$ "generated" at $\mathbf{x}_A$ by our collection of charges, which for this simple case is just a single particle of charge q_B located at $\mathbf{x}_B$, i.e.

$$\vec{F}_A = \frac{q_A q_B}{4\pi\epsilon_o |\vec{x}_A - \vec{x}_B|^2} \cdot \frac{\vec{x}_A - \vec{x}_B}{|\vec{x}_A - \vec{x}_B|} = q_A \vec{E}(\vec{x}_A)$$



We can just divide the middle and right terms by q_A to obtain

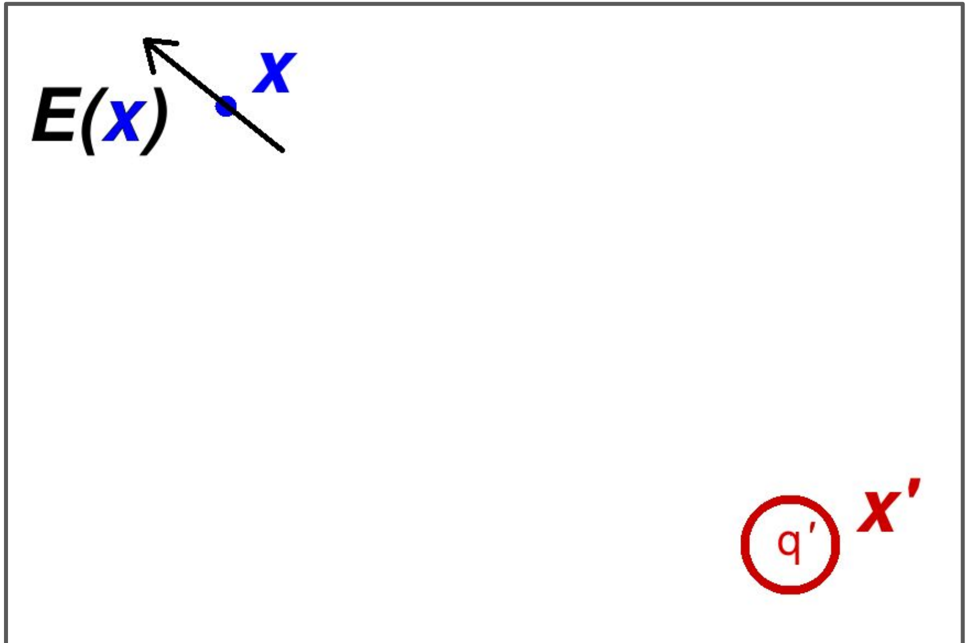
$$\vec{E}(\vec{x}_A) = \frac{q_B}{4\pi\epsilon_o |\vec{x}_A - \vec{x}_B|^2} \cdot \frac{\vec{x}_A - \vec{x}_B}{|\vec{x}_A - \vec{x}_B|}$$

Main point: Electric field of single point charge

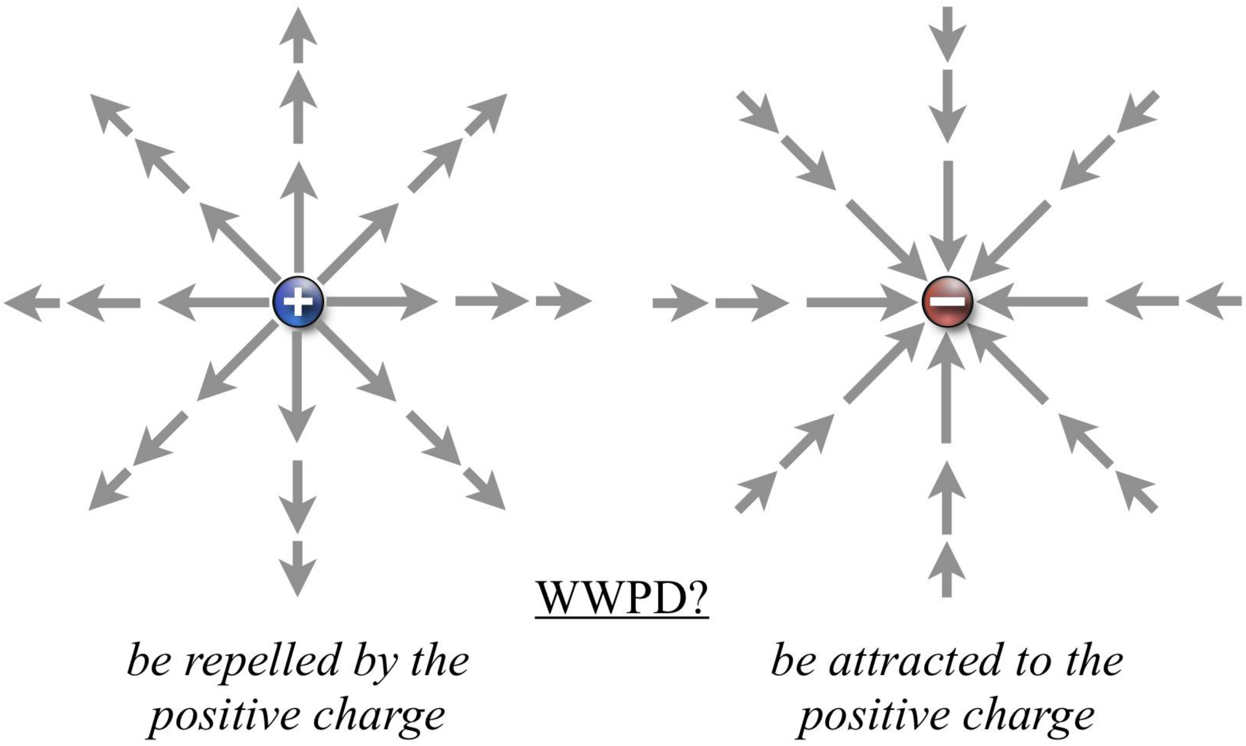
Cleaning up our notation a bit, let us state definitively:

The electric field **E**(***x***) at a point ***x*** generated by a particle of charge *q'* located at point ***x'*** is given by

$$\vec{E}(\vec{x}) = \frac{q'}{4\pi\epsilon_o|\vec{x}-\vec{x}'|^2} \cdot \frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|}$$



Visualizing electric field of a point charge:
"what would a proton do?" (WWPD)



https://physics.nfshost.com/textbook/04-ElectricFields/img/wwpd.png

Main point: superposition principle + electric field

To determine the electric field due to multiple (not just one) “point charges” (= charged particles), by the superposition principle we can simply “add together” the electric fields generated by each point charge

e.g. if $\mathbf{E}_B(\mathbf{x})$ and $\mathbf{E}_C(\mathbf{x})$ are the electric fields generated by point charges *B* and *C* in the absence of one another, then the electric field generated when both charges are present is simply

$$\mathbf{E}_{B,C}(\mathbf{x}) = \mathbf{E}_B(\mathbf{x}) + \mathbf{E}_C(\mathbf{x})$$

Quiz time!

two point charges, equal and negative, are placed as shown in the diagram to the right.

Instapoll question: what is the direction of the electric field at the point **P**?

(hint: WWPD?)