302L S20 HW#6 Solutions

1. The capacitance of a dielectric filled parallel plate capacitor is given by equation 2 in section 3 of lecture 12:
$$C = \frac{\epsilon_r \epsilon_o A}{d}$$
The dielectric in this instance is the 50 nm thick alumina layer. Alumina has a relative permittivity $\epsilon_r = 10$. The area of the plates is
$$A = 1\,\mathrm{cm} \times 5\,\mathrm{cm} = 5 \times 10^{-4}\,\mathrm{m}^2$$
The separation d of the plates is the 50 nm thickness of the alumina layer, since its is surrounded on both sides by conductors (the aluminum foil on one side, and the electrolyte on the other). Plugging these in we get
$$C = \frac{10 \times 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}} \times 5 \times 10^{-4}\,\mathrm{m}^2}{50 \times 10^{-9}\,\mathrm{m}} \approx 1\,\mathrm{\mu F}$$

2. In commercial electrolytic capacitors the aluminum foils are electrochemically roughened to increase the effective plate area A and thus the capacitance. The increase can be very significant (up to 200x). See section 4 of lecture 12.

3. This problem is the same as question 1 except we substitute
   - $A = 10\,\mathrm{mm}^2 = 10 \times 10^{-6}\,\mathrm{m}^2$
   - $d = 5\,\mathrm{nm}$
   - $\epsilon_r = 5$

   Plugging these in we get
$$C = \frac{5 \times 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}} \times 10 \times 10^{-6}\,\mathrm{m}^2}{5 \times 10^{-9}\,\mathrm{m}} \approx 1\,\mathrm{\mu F} \approx 89\,\mathrm{nF}$$

4. The capacitance of each eel electrocyte is just $\frac{11.3}{10}$ times the capacitance of the ray electrocyte capacitance, since capacitance is proportional to plate area. i.e.
$$C = 1.13 \times 89\,\mathrm{nF} \approx 100\,\mathrm{nF}$$
From the equation for the energy of a capacitor
$$E = \frac{1}{2} C V^2$$
we find that the energy of each electrocyte when it is charged is given by
$$E = \frac{1}{2} 100 \times 10^{-9}\,\mathrm{F} \times (0.1\,\mathrm{V})^2 = 0.5\,\mathrm{nJ}$$
The energy E_T of the whole electric organ is just the energy of one electrocyte times the number of electrocytes, i.e.
$$E_T = 35 \times 6000 \times 0.5\,\mathrm{nJ} \approx 100\,\mathrm{\mu J}$$
This is more than a million times smaller than the 150 J of energy stored in a typical defibrillator.

5. From equation 3 in section 5.1 of lecture 12 we have

$$q\delta = \alpha E$$

Solving for δ we have

$$\delta = \alpha E/q$$

plugging in

- $q = 18e$
- $E = 1\,\mathrm{MV\,m^{-1}} = 1 \times 10^{6}\,\mathrm{V\,m^{-1}}$
- $\alpha = 1 \times 10^{-11} \frac{e\text{Å}}{V/m}$

we get

$$\delta = \frac{1 \times 10^{-11} \frac{e\text{Å}}{V/m} \times 1 \times 10^{6}\,\mathrm{V/m}}{18e} \approx 5 \times 10^{-7}\,\text{Å} = 0.05\,\mathrm{fm}$$

This is more than a hundred times smaller than the nucleus of the argon atom!

6. From equation 6 of section 5.2 of lecture 12 we have

$$\epsilon_r = 1 + \frac{n\alpha}{\epsilon_o}$$

solving for α we get

$$\alpha = \frac{(\epsilon_r - 1)\,\epsilon_o}{n}$$

To determine n the number of neon atoms per unit volume, we recall from basic chemistry that one mole of gas at STP occupies $22.4\mathrm{L} = 0.0224\,\mathrm{m}^3$. Therefore the density n neon atoms at STP is

$$n = \frac{1\,\mathrm{mol}}{0.0224\,\mathrm{m}^3} = \frac{6.02 \times 10^{23}\,\mathrm{atoms}}{0.0224\,\mathrm{m}^3} = 2.69 \times 10^{25}\,\mathrm{atoms/m^3}$$

plugging this in we get

$$\alpha = \frac{1.3 \times 10^{-4} \times 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}}{2.69 \times 10^{25}\,\mathrm{atoms/m^3}} \approx 3 \times 10^{-12} \frac{e\text{Å}}{V/m}$$