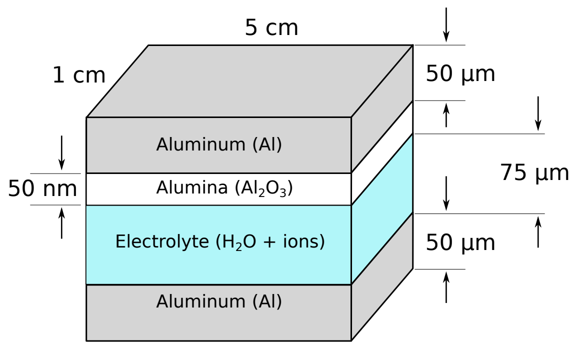
302L S20 Homework 6 - Dielectrics

Note: the order of the question choices on Quest are automatically randomized and so may be different from the order on this document.

Question 1 (part 1 of 2):

An electrolytic capacitor is made from two thin rectangular sheets of aluminum foil. The sheets both have dimensions of 1cm x 5cm x 50 µm. The two sheets are spaced apart by 50 µm and the space between the sheets is filled with electrolytic fluid. One of the sheets contains a 50nm thick layer of alumina (Al_2O_3). Here is a diagram of the capacitor:



Assuming the sheets are perfectly flat, what is the capacitance of this capacitor? Pick the answer closest to your calculated result:

(a) 0.1 nF

(b) 1.0 nF

(c) 0.1 µF

(d) 1.0 µF

Hint: Refer to section 4 of the lecture 12 notes. What is the effective plate spacing d of this capacitor?

Question 2 (part 2 of 2):

A commercial aluminum electrolytic capacitor containing all the same dimensions as the one described in the previous problem is found to have a capacitance that is roughly 100 times larger than the value we found in the previous problem. Why is this?

(a) The commercial capacitor does not use flat aluminum foil but instead uses aluminum foil that is roughened.

(b) The commercial capacitor does not use pure aluminum foil but instead uses an aluminum alloy that contains copper.

(c) The commercial capacitor does not have a pure alumina dielectric layer but instead has an alumina layer that is intentionally contaminated ("doped") with chromium impurities.

(d) The commercial capacitor does not use a water-based electrolyte but instead uses a solid electrolyte.

Question 3 (part 1 of 2):

In order to store a lot of charge, the electrocytes of electric fish have evolved to be have a large surface area. Suppose that the surface area of the electrocytes of the electric ray are $10\,\text{mm}^2$. (Compare this to typical skin cell or red blood cell, which have surface areas on the order of $0.05\,\text{mm}^2$) Given that the electrocyte's membrane has a thickness of of $5\,\text{nm}$, and that the lipid bilayer composing the cell membrane has a relative permittivity of 5, what is the capacitance of an electrocyte? Pick the answer that is closest to your calculated result:

(a) 17 pF

(b) 89 pF

(c) 17 nF

(d) 89 nF

(e) 17 µF

(f) 89 µF

Question 4 (part 2 of 2)

The electric organ of the average electric eel contains 35 stacks of electroctyes, where each stack contains roughly 6,000 electrocytes. Suppose that the electrocytes of the electric eel are identical to the electrocytes of the electric ray, except that the surface area of the eel's electrocytes are $11.3\,\text{mm}^2$. Given that the average membrane potential of an electocyte is about 100 mV, estimate the energy stored in an electric eel's in a fully charged electric organ. Pick the answer that is closest to your calculated result:

(a) 100 nJ

(b) 100 µJ

(c) 100 mJ

(d) 100 J

How does this energy compare to the energy stored in a defibrillator (150 J)?

Hint: The stored energy does not depend on the arrangement of the electrocytes, only their total number.

Question 5:

Liquid argon has a density of $2.1 \times 10^{28}\,\text{atoms/m}^3$ and a relative permeability ϵ_r of 1.325. Using equation 6 from the lecture 12 notes, we get an estimate of $\alpha \approx 1 \times 10^{-11} \frac{e\text{Å}}{V/m}$ for the polarizability α of the argon atom. Note that e is the charge of a proton, and $1\text{Å} = 1 \times 10^{-10}\,\text{m}$ (called an *angstrom*). Use equation 3 from section 5.1 of lecture 12 to determine the maximum separation δ of the electron cloud and nucleus of an argon atom during a thunderstorm, where the electric field can be as large as $1\,\text{MV/m}$. Note that the nucleus of the argon atom has a nuclear charge $q = 18e$. Pick the answer closest to your calculated answer:

(a) $0.05\,\text{fm} = 0.05 \times 10^{-15}\,\text{m}$

(b) 0.05 pm

(c) 0.05 nm

(d) 0.05 µm

For comparison, the size of the argon atom itself is about 100 pm (i.e. 1Å), and the size of the argon nucleus is almost 10 fm.

Question 6:

Neon gas at standard temperature and pressure has a relative permittivity ϵ_r of 1.00013. Use the ideal gas law $PV = nkT$ and equation 6 of section 5.2 of lecture 12 to compute the polarizability α of a neon atom. Give you answer in units of $\frac{e\text{Å}}{\text{V}/m}$. Pick the answer that is closest to your calculated result:

(a) $3 \times 10^{-18} \frac{e\text{Å}}{\text{V/m}}$

(b) $3 \times 10^{-12} \frac{e\text{Å}}{\text{V/m}}$

(c) $3 \times 10^{-6} \frac{e\text{Å}}{\text{V/m}}$

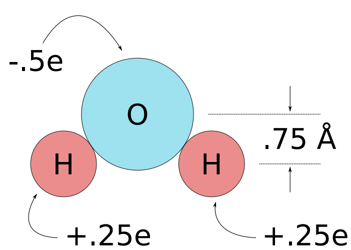
(d) $3 \frac{e\text{Å}}{\text{V/m}}$

How does your answer compare to the value given in the graph in section 5.1 of lecture 12?

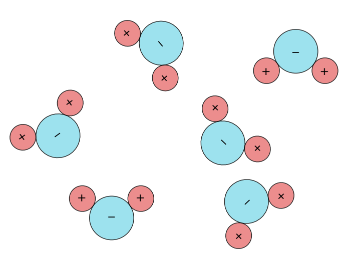
Question 7 (extra credit!):

In lecture 12 we learned that the relative permittivity ϵ_r of a material is determined by the polarizability α of the atoms making up the material. Conceptually, we found an electric field inside the dielectric caused the atoms in the dielectric to polarize, and that this polarization caused partial screening of the charge on the plates of the capacitor. This charge screening, in turn, resulted in an increase in the capacitance, and this increase is reflected in the dielectric's relative permittivity ϵ_r.

This picture, however, applies only to the case of *non-polar* materials, where the negative charge clouds of the atoms are perfectly centered around their associated nuclei when there is no electric field. In a *polar* material like water (H_2O), the molecules making up the material have a non-zero dipole moment, even when there is no electric field applied. For example, in a water molecule, about a quarter of the charge of each hydrogen atom is transferred to the oxygen atom:

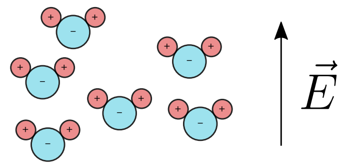


We say then that the water molecule has a *permanent* dipole moment p equal to roughly $.5e \times .75\text{Å} = .385\,e\text{Å}$. (Compare this to the induced dipole moment of the argon atom in a thunderstorm you calculated in Question 6.) When there is no electric field, there is no preference for the molecules to point in any particular direction:



With zero electric field the average dipole moment p_{avg} of the water molecules is zero, since for every water molecule pointing one direction there is another point in the opposite direction.

However, when a very strong electric field is applied, these permanent dipoles *align* so that their positive tail points towards the electric field:



If all the molecules have exactly the same alignment as they do in the above illustration, then we would say that the average dipole moment p_{avg} is equal to p, the permanent dipole moment of the polar molecule.

By measuring the capacitance of a water filled capacitor, it is determined that, *at room temperature*, the average dipole moment p_{avg} of a water molecule in an strong electric field of strength $E = 1\,\text{kV}\,\text{mm}^{-1}$ is $.0016\,e\text{Å}$, which is only small fraction of the water molecule's permanent dipole moment ($.385e\text{Å}$).

Why is this? Give an answer 1-2 paragraphs in length.

To gain you some insight into the question, navigate to[1]:

https://creilly.github.io/h2o.html

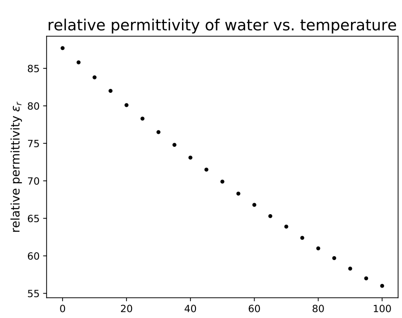
The website shows a simulation of a polar dielectric consisting of 20 molecules that have been placed between the two plates of a capacitor. You can control the strength of the electric field between the plates and the temperature of dielectric. The displayed "alignment" number gives the ratio $\frac{p_{\text{avg}}}{p}$ of the average dipole moment to the permanent dipole moment.

Some questions to consider:

1. Set the electric field to be very high and the temperature to be very low. What do you observe? What is the alignment ratio? Can you tell which direction the electric field is pointing (up or down)?

2. Now set the electric field to be low and the temperature high. What happened to the alignment ratio? Why?

Are your conclusions consistent with the following plot of the dielectric constant ϵ_r of water versus temperature?



[1]The site works alright on a smart phone but I've found much smoother performance on PCs and laptops.